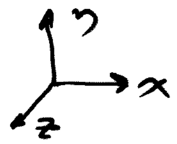
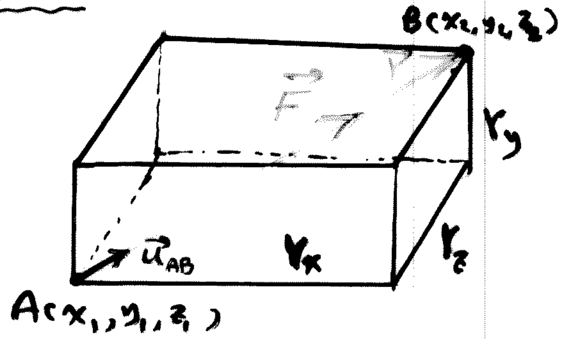
Force Directed along a Line (3-D)

$\vec{r} = \vec{AB} = B(\ldots) - A(\ldots)$ ⟸ Position Vector

$= r_x \vec{i} + r_y \vec{j} + r_z \vec{k}$

$= (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k}$





$\vec{u}_r = \dfrac{\vec{r}}{r}$

$= \dfrac{1}{r}(r_x \vec{i} + r_y \vec{j} + r_z \vec{k})$

$\vec{u}_F = \dfrac{\vec{F}}{F}$

Since $\vec{F}$ and $\vec{r}$ are on the <u>same</u> line, $\boxed{\vec{u}_r = \vec{u}_F}$

$$\Rightarrow \quad \vec{F} = F\vec{u}_F$$
$$= F\vec{u}_r$$
$$= F\left[\frac{1}{r}(r_x\vec{i} + r_y\vec{j} + r_z\vec{k})\right] = \frac{F}{r}(r_x\vec{i} + r_y\vec{j} + r_z\vec{k})$$

Thus:

$$F_x = \frac{F}{r} r_x = \frac{r_x}{r} F$$

$$F_y = \frac{r_y}{r} F$$

$$F_z = \frac{r_z}{r} F$$

$$\Rightarrow \quad \cos\theta_x = \frac{F_x}{F} = \frac{\frac{r_x}{r}F}{F} = \frac{r_x}{r}$$

$$\cos\theta_y = \frac{r_y}{r}$$

$$\cos\theta_z = \frac{r_z}{r}$$

$$\vec{R} = (\Sigma F_x)\vec{i} + (\Sigma F_y)\vec{j} + (\Sigma F_z)\vec{k}$$
$$= R_x\vec{i} + R_y\vec{j} + R_z\vec{k}$$

For the direction of the resultant:

$$\cos\theta_x = \frac{R_x}{R} \; ; \quad \cos\theta_y = \frac{R_y}{R} \; ; \quad \cos\theta_z = \frac{R_z}{R}$$

* Note that graphical or trigonometric methods are <u>NOT</u> practical in 3-D