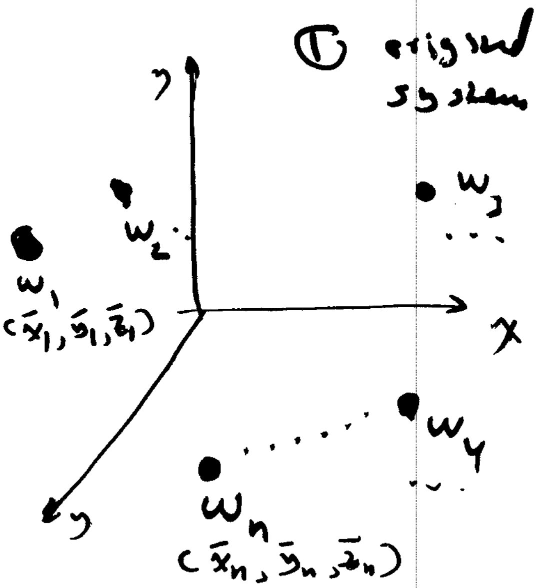
Center of Gravity

Centroid

To replace the system of particles / bodies by a simple system composed of only one body, two conditions must be satisfied

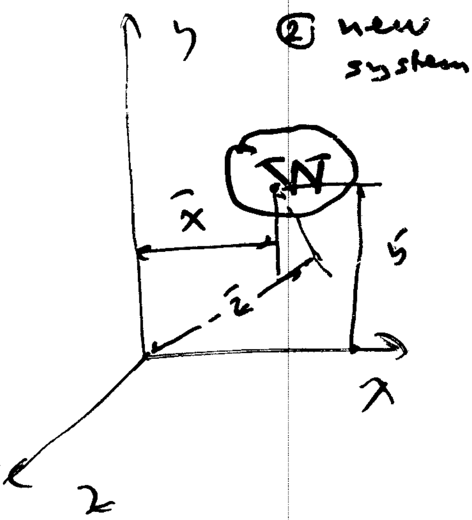


((as far as Statics is concerned)). Thus the two systems will be equivalent.

① Σ Forces in the two systems must be equal. ⇒

ΣF_y :

$$\bar{W} = \sum_{i=1}^{n} w_i$$



② Σ Moments in the two systems must be equal.

ΣM_x :

$$\bar{z}\,\bar{W} = \sum_{i=1}^{n} \bar{z}_i w_i$$

$$\Rightarrow \quad \bar{z} = \frac{\sum_{i=1}^{n} \bar{z}_i w_i}{\underbrace{\bar{W}}_{\Sigma w_i}}$$

similarly :

$$M_z \Rightarrow \quad \bar{x} = \frac{\sum_{i=1}^{n} x_i w_i}{\bar{W}}$$

If w_i gets smaller and smaller, then $w_i \to \Delta w_i \to dw_i$

$\Rightarrow \quad \bar{W} = \sum (\Delta w)_i \Rightarrow \bar{W} = \int dw$; similarly, $\bar{x} = \frac{\int \bar{x}_i\, dw}{\int dw}$, $\bar{z} = \dots$

Centroid

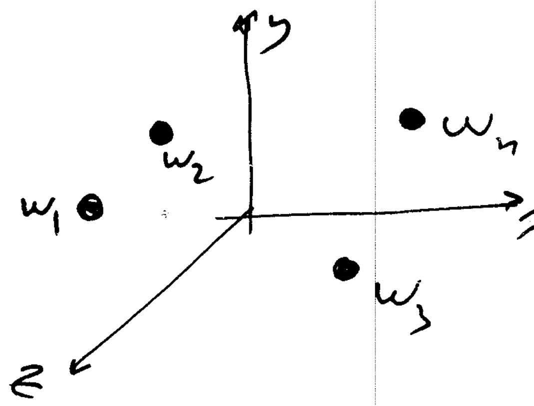
C. for "Particles"

$$\bar{x}\sum w_i = \sum_{i=1}^{n} \bar{x}_i w_i$$

$$\Rightarrow \quad \bar{x} = \frac{\sum \bar{x}_i w_i}{\sum w_i}$$

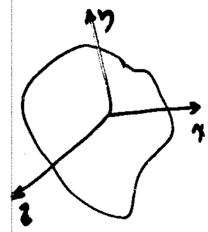
$$\bar{y} = \ldots$$

$$\bar{z} = \ldots$$



C.G. for bodies:

$$\bar{x} = \frac{\int \bar{x}\, dw}{\int dw} \quad ; \quad \bar{y} = \ldots, \quad \bar{z} = \ldots$$



C.M.

$$\bar{x} = \frac{\int \bar{x}\, \rho\, dv}{\int \rho\, dv} \quad ; \quad \bar{y} = \ldots, \quad \bar{z} = \ldots$$

<u>Centroid</u>

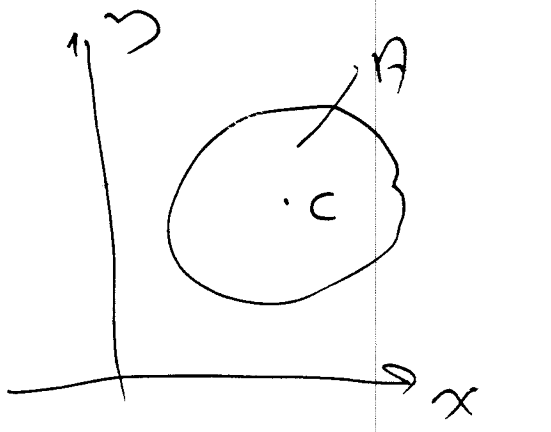
$$\bar{x} = \frac{\int_V \bar{x}\, dV}{\int_V dV}, \quad \bar{y} = \ldots, \quad \bar{z} = \ldots \quad \Leftarrow V$$

$$\bar{x} = \frac{\int_A \bar{x}\, dA}{\int_A dA} \quad \ldots \quad \Leftarrow A$$

$$\bar{x} = \frac{\int_L \bar{x}\, dL}{\int_L dL} \quad \ldots \quad \Leftarrow L$$

$$\bar{x} A = \int x \, dA$$

$$\bar{y} A = \int y \, dA$$



$\int x \, dA$ = first moment of the area wrt y-axis

$\int y \, dA$ = " " " " " x-"

If the centroid of an area is located on a coordinate axis, then the first moment of the area wrt that axis is ZERO

Symmetry $\Longleftarrow$ Take Advantage of

Centroid for Composite Bodies
Volumes
Areas

We can come back to the original definition. $\Rightarrow$

use $\boxed{\Sigma \Delta}$ instead of $\boxed{\int}$. $\Rightarrow$

$$\bar{x} = \frac{\Sigma \bar{x}_i V_i}{\Sigma V_i} \qquad \bar{x} = \frac{\Sigma \bar{x}_i A_i}{\Sigma A_i} \qquad \cdots\cdots$$

$$\vdots \qquad\qquad\qquad \vdots$$