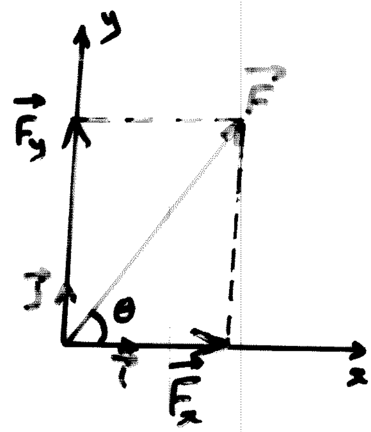
Cartesian Vectors (2-D)

Cartesian (rectangular) components of forces:

F_x and F_y are called the rectangular or Cartesian components of $\vec{F}$.



Let's introduce the unit vectors $\vec{i}$ in the x-direction and $\vec{j}$ in the y-direction. $\Rightarrow$

$$\vec{F} = \vec{F_x} + \vec{F_y}$$
$$= F_x \vec{i} + F_y \vec{j}$$

$\vec{F_x}$ and $\vec{F_y}$ are the vector components of $\vec{F}$, while F_x and F_y are the scalar components of $\vec{F}$.

$$\left.\begin{aligned} F_x &= F \cos\theta \\ F_y &= F \sin\theta \\ \tan\theta &= \frac{F_y}{F_x} \end{aligned}\right\}$$

Note θ between $\underline{\underline{F}}$ and $\underline{\underline{X}}$.

How to add forces:

$$\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \cdots + \vec{F_n}$$
$$= \sum_{i=1}^{n} \vec{F_i}$$

Thus,

$$\vec{R} = (F_{1x}\vec{i} + F_{1y}\vec{j}) + \cdots + (F_{nx}\vec{i} + F_{ny}\vec{j})$$
$$= (F_{1x} + F_{2x} + \cdots F_{nx})\vec{i} + (F_{1y} + F_{2y} + \cdots + F_{ny})\vec{j}$$

Follow the following steps:

① Calculate the x and y components of each force.

② Add the x and y components of all the forces.

$$\vec{R} = (\Sigma F_x)\vec{i} + (\Sigma F_y)\vec{j}$$
$$= R_x\vec{i} + R_y\vec{j}$$

③ Compute R:

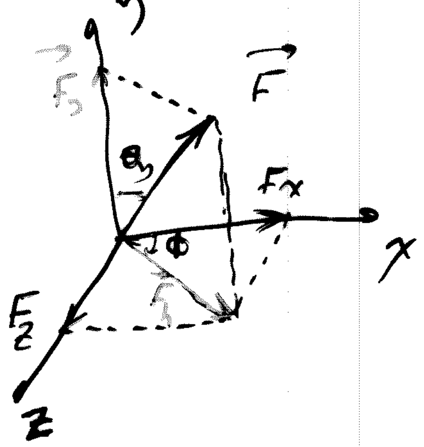
$$R = \sqrt{R_x^2 + R_y^2}$$

$$\tan\theta_R = \frac{R_y}{R_x}$$

Cartesian Vector in 3-D

3. ~~Determining components of a vector parallel and normal to a given direction.~~ ([redacted])

①



$$F_y = F\cos\theta_y$$

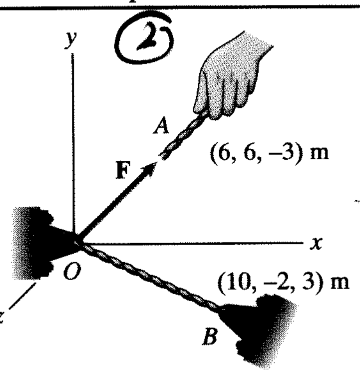
$$F_h = F\sin\theta_y$$

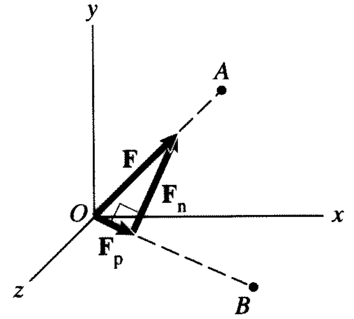
$\Downarrow$

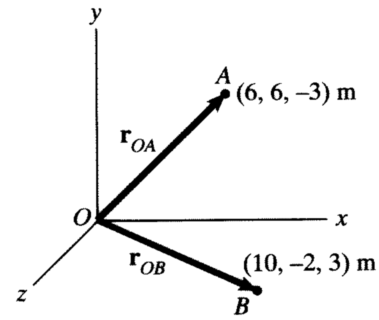
$$F_x = F_h\cos\phi = F\sin\theta_y\cos\phi$$

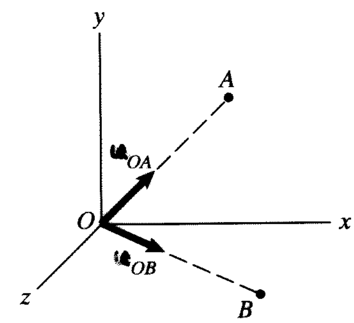
$$F_z = F_h\sin\phi = F\sin\theta_y\sin\phi$$

②

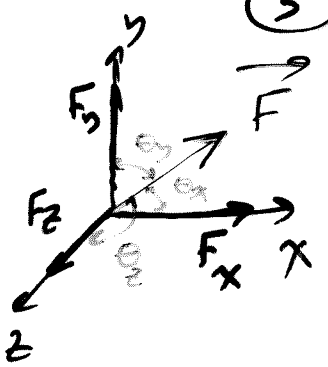








③



$$\vec F = \vec F_x + \vec F_y + \vec F_z$$
$$= F_x\vec i + F_y\vec j + F_z\vec k$$
$$= F\cos\theta_x\vec i + F\cos\theta_y\vec j + F\cos\theta_z\vec k$$
$$= F(\cos\theta_x\vec i + \cos\theta_y\vec j + \cos\theta_z\vec k)$$

$$|\vec F| = F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$\Rightarrow F^2 = (F\cos\theta_x)^2 + (F\cos\theta_y)^2 + (F\cos\theta_z)^2$$

$$F^2 = F^2(\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z)$$

$$\Rightarrow \boxed{\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1}$$

Recall:

$$\vec F = F\vec u$$

$\vec u$ is the unit vector along F

$$\Rightarrow \boxed{\vec u = \cos\theta_x\vec i + \cos\theta_y\vec j + \cos\theta_z\vec k}$$

$\Uparrow$

Do NOT forget!!

$$\vec R = \Sigma\vec F_i$$
$$= (\Sigma F_x)\vec i + (\Sigma F_y)\vec j + (\Sigma F_z)\vec k$$
$$= R_x\vec i + R_y\vec j + R_z\vec k$$