Equilibrium of Rigid Bodies

For a rigid body to be in equilibrium, the sum of the external forces must be zero. ⟹

$$\boxed{\sum \vec{F} = \vec{0}}$$

$$\Rightarrow \quad (\Sigma F_x)\,\vec{i} + (\Sigma F_y)\,\vec{j} + (\Sigma F_z)\,\vec{k} = 0$$

$$\Rightarrow \quad \Sigma F_x = 0 \qquad (1)$$

$$\Sigma F_y = 0 \qquad (2)$$

$$\Sigma F_z = 0 \qquad (3)$$

The equations above are necessary and sufficient for a particle. However, for a rigid body, these equations are necessary but NOT sufficient. For example, if there is a couple applied on a rigid body, then $\Sigma F = 0$, but the body can rotate due to the moment of the couple; thus to have "complete" equilibrium, the following equation must be satisfied also:

$$\boxed{\sum \vec{M} = \vec{0}}$$

$$\Rightarrow \quad \Sigma M_x = 0 \qquad (4)$$

$$\Sigma M_y = 0 \qquad (5)$$

$$\Sigma M_z = 0 \qquad (6)$$

In **2-D**,

$\Sigma F_x = 0$ ① ; $\Sigma F_y = 0$ ② , $\Sigma M = 0$ ③ .
($\uparrow M_z$)

* 3 eq. & 3 unknown max.

Free Body Diagram (FBD)

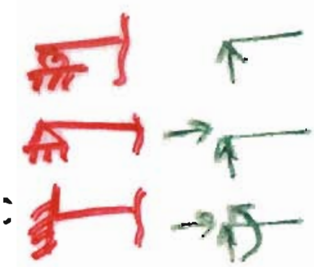
FBD FBD FBD FBD

In the FBD, you must include **all** forces, not more, not less !!!

Forces:
- known : weight, applied external force.
- unknown : reactions

Supports & Reactions :

① one force / unknown reaction :
② two force(s) / unknown(s) reactions :
③ three force(s) / unknown(s) reactions :



See complete table next. (Also your text, pp. 196-

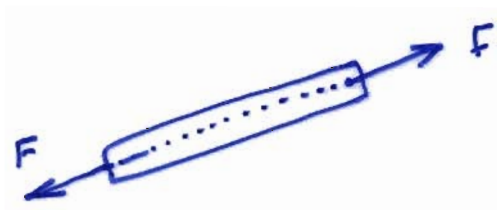
* Alternative Equations:

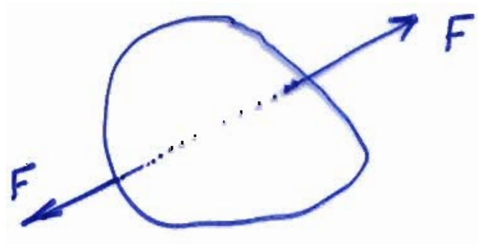
$\Sigma F_x = 0$; $\Sigma M_1 = 0$; $\Sigma M_2 = 0$
$\Sigma F_y = 0$; $\Sigma M_1 = 0$; $\Sigma M_2 = 0$
$\Sigma M_1 = 0$; $\Sigma M_2 = 0$; $\Sigma M_3 = 0$

} "Appropriate" poi… must be taken fo… the moment.

* Note that you can NOT use more than 3 equations. Why ?!!

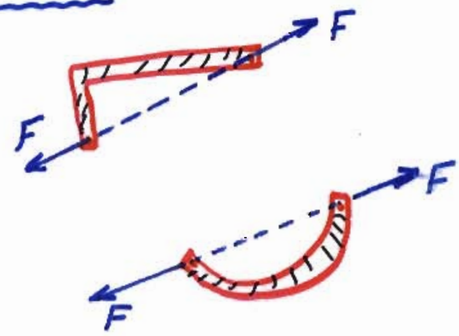
* Two-Force members :





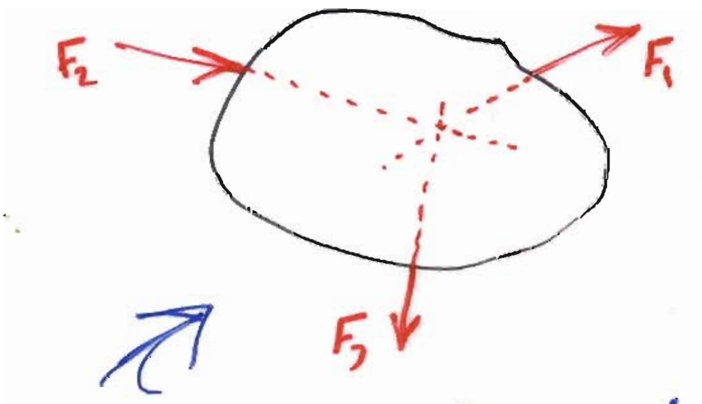
If only two forces are applied on a body, then equilibri
can NOT be maintained unless the two forces are :

① equal
② on the same line of action
③ opposite sense.

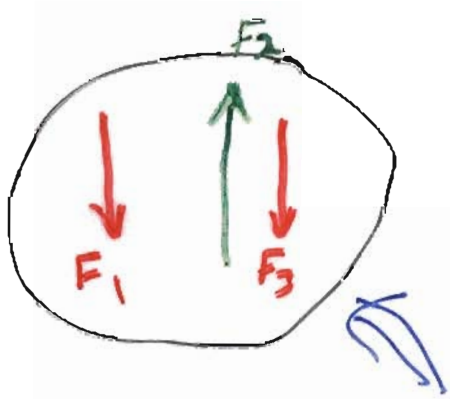


Try any other possibilities !!
⇒ You can not have equilibrium !

* Three-Force members :



The three forces meet at one point



The three forces ar
parallel with the
force in the middl
F_2 is the opposite
direction of the oth
two F_1 and F_3. Als
$F_2 = F_1 + F_3$

The two cases above are the only possibilites if equilibrium is to be maintained. Try others !