

Equilibrium of Rigid Bodies 1/3

For a rigid body to be in equilibrium, the sum of the external forces must be zero. \Rightarrow

$$\boxed{\sum \vec{F} = \vec{0}}$$

$$\Rightarrow (\sum F_x) \vec{i} + (\sum F_y) \vec{j} + (\sum F_z) \vec{k} = 0$$

$$\Rightarrow \sum F_x = 0 \quad \text{①}$$

$$\sum F_y = 0 \quad \text{②}$$

$$\sum F_z = 0 \quad \text{③}$$

The equations above are necessary and sufficient for a particle. However, for a rigid body, these equations are necessary but NOT sufficient. For example, if there is a couple applied on a rigid body, then $\sum F = 0$, but the body can rotate due to the moment of the couple; thus to have "complete" equilibrium, the following equation must be satisfied also:

$$\boxed{\sum \vec{M} = \vec{0}}$$

$$\Rightarrow \sum M_x = 0 \quad \text{④}$$

$$\sum M_y = 0 \quad \text{⑤}$$

$$\sum M_z = 0 \quad \text{⑥}$$

In 2-D,

$\Sigma F_x = 0$ ① ; $\Sigma F_y = 0$ ② , $\Sigma M = 0$ ③
* 3 eq & 3 unknown max. ↑ M_z

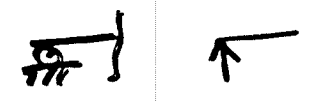
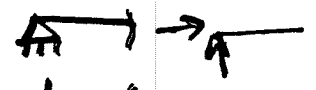
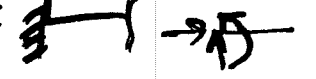
Free Body Diagram (FBD)

FBD FBD FBD FBD

In the FBD, you must include all forces, not more, not less !!!

Forces $\left\{ \begin{array}{l} \text{known: weight, applied external forces} \\ \text{unknown: reactions} \end{array} \right.$

Supports & Reactions:

- ① one force / unknown reaction : 
- ② two force(s) / unknown(s) reactions : 
- ③ three force(s) / unknown(s) reactions : 

See complete table next. (Also your text, pp 196-97)

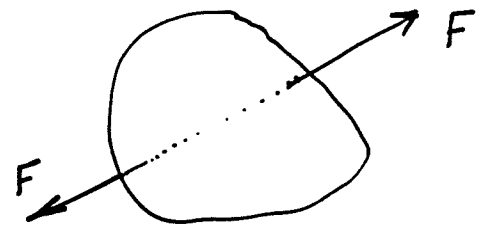
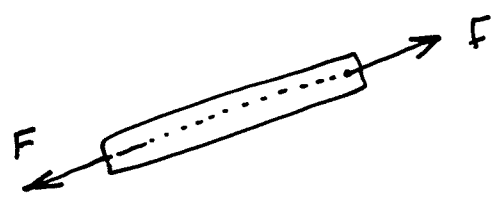
* Alternative Equations:

$$\left. \begin{array}{l} \Sigma F_x = 0 ; \Sigma M_1 = 0 ; \Sigma M_2 = 0 \\ \Sigma F_y = 0 ; \Sigma M_1 = 0 ; \Sigma M_2 = 0 \\ \Sigma M_1 = 0 ; \Sigma M_2 = 0 ; \Sigma M_3 = 0 \end{array} \right\}$$

"Appropriate" points must be taken for the moment.

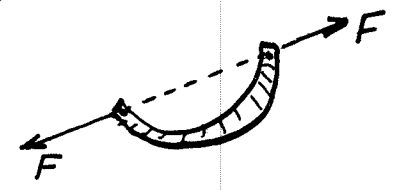
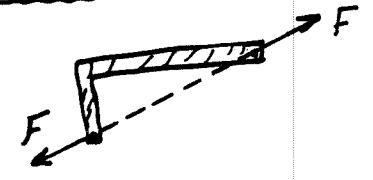
* Note that you can NOT use more than 3 equations. Why??

* Two-Force members :



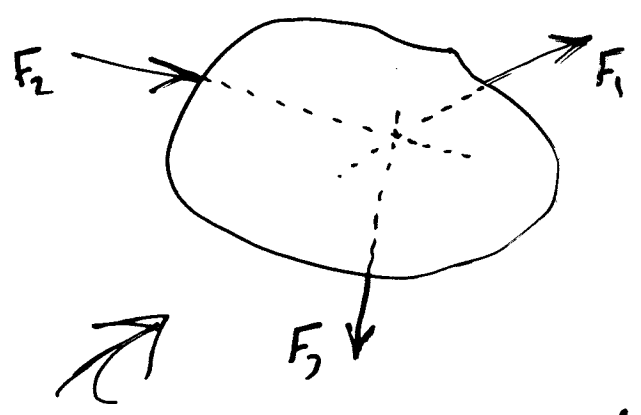
If only two forces are applied on a body, then equilibrium can NOT be maintained unless the two forces are :

- ① equal
- ② on the same line of action
- ③ opposite sense.

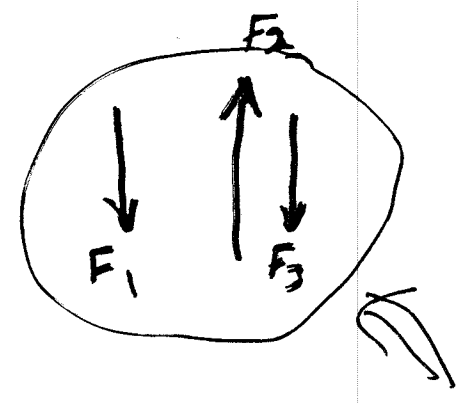


Try any other possibilities !!
 ⇒ You can not have equilibrium !

* Three-Force members :



The three forces meet
at one point



The three forces are parallel with the force in the "middle" F₂ is in the opposite direction of the other two F₁ and F₃. Also $F_2 = F_1 + F_3$

The two cases above are the only possibilities if equilibrium is to be maintained. Try others !