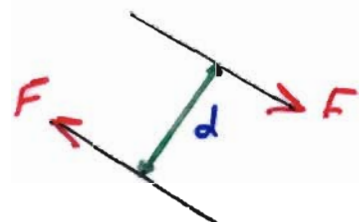
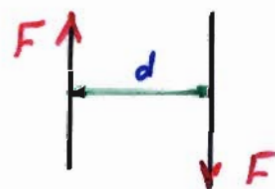


Moment of a Couple

* Couple :

- ① two forces
- ② same magnitude
- ③ parallel lines of action
- ④ opposite sense



$$\Sigma F = 0 \text{ in any direction}$$

$$\Sigma M = \pm Fd \text{ about any point}$$

↓
Try in 2-D ⇒ OK!!

Take 3-D in general :

$$\begin{aligned} \Sigma \vec{M}_O &= (\vec{r}_A \times \vec{F}) + \vec{r}_B \times (-\vec{F}) \\ &= (\vec{r}_A - \vec{r}_B) \times \vec{F} \end{aligned}$$

From the figure,

$$\vec{r} = \vec{r}_A - \vec{r}_B$$

$$\Rightarrow \vec{M} = \vec{r} \times \vec{F}$$

Note that \vec{r} is independent of O !

↳

Since $\Sigma \vec{F} = 0$ in any direction
and $\vec{M} = \vec{r} \times \vec{F}$ about any point

⇒ \vec{M} is a free vector.

⇒ Only the moment of the couple, not the couple which counts.

⊛ Different couples acting on the same plane or parallel planes may have equal moments. ⇒ They will have the same effect on the rigid body.

$$\vec{M}_R = \Sigma \vec{M}_i = \Sigma (\vec{r}_i \times \vec{F}_i)$$

