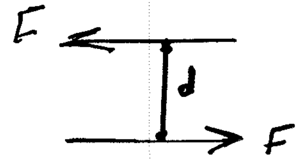
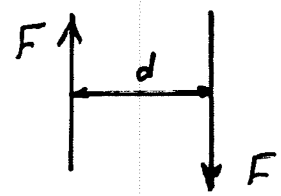
Moment of a Couple

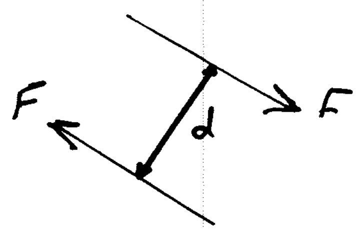
* **Couple** :



① two forces
② same magnitude
③ parallel lines of action
④ opposite sense



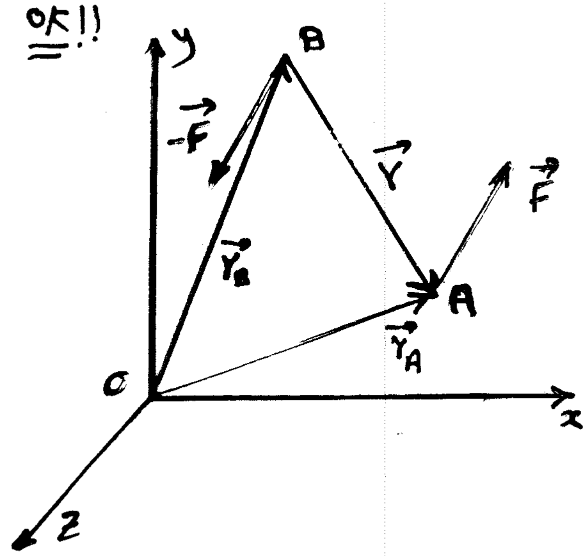
$\Sigma F = 0$ in any direction



$\Sigma M = \pm Fd$ about any point

⇓

Try in 2-D ⇒ OK!!



Take 3-D in general :

$$\Sigma \vec{M}_0 = (\vec{r}_A \times \vec{F}) + \vec{r}_B \times (-\vec{F})$$

$$= (\vec{r}_A - \vec{r}_B) \times \vec{F}$$

From the figure,

$$\vec{r} = \vec{r}_A - \vec{r}_B$$

$$\Rightarrow \quad \vec{M} = \vec{r} \times \vec{F}$$

Note that $\vec{r}$ is independent of O !

⇒

Since $\Sigma \vec{F} = 0$ in any direction
and $\vec{M} = \vec{r} \times \vec{F}$ about any point

⇒ $\vec{M}$ is a free vector.

⇒ Only the moment of the couple, not the couple which counts.

⊛ Different couples acting on the same plane or parallel planes may have equal moments. ⇒ They will have the same effect on the rigid body.

$$\boxed{\vec{M}_R = \Sigma \vec{M}_i = \Sigma (\vec{r}_i \times \vec{F}_i)}$$