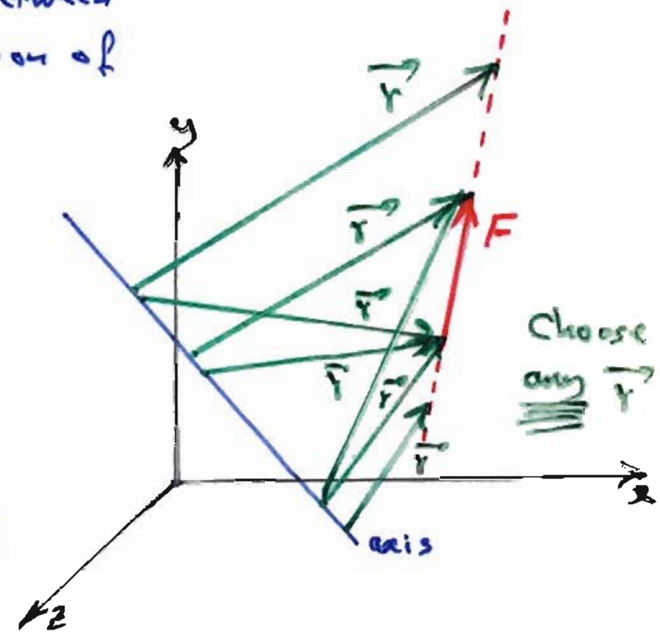
Moment about an Axis

1) Scalar Analysis:

If the perpendicular distance d between the axis and the line of action of $\vec{F}$ is known, then

$$M_{axis} = F\,d$$



Choose any $\vec{r}$

2) Vector Analysis:

Most of the time, d is not known; thus, vector analysis is utilized.

First, let's find the moment of the force about any point on the axis, say P:

$$\Rightarrow \quad \vec{M}_P = \vec{r} \times \vec{F}$$

This was discussed in the previous topic. Now remember that $\vec{r}$ is the a position vector going from any point on the axis to any point on the line of action of $\vec{F}$.

Second, we can think of $\vec{M}_P$ as any vector; we can find the projection (component) of $\vec{M}_P$ on that axis using the Dot Product discussed earlier. $\Rightarrow$

$$M_{axis} = \vec{u}_{axis} \cdot \vec{M}_P \quad \leftarrow \text{[scalar]} \leftrightarrow \text{Volume !!}$$

$$= \vec{u}_{axis} \cdot (\vec{r} \times \vec{F})$$

$$= (u_x\vec{i} + u_y\vec{j} + u_z\vec{k}) \cdot (M_x\vec{i} + M_y\vec{j} + M_z\vec{k})$$

This is called mixed triple product [$\cdot$ and $\times$] with three vectors $\vec{u}, \vec{r}, \vec{F}$.

or

$$\boxed{M_{axis} = \begin{vmatrix} u_x & u_y & u_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}}$$

In vector form:

$$\vec{M}_{axis} = M_{axis}\,\vec{u}_{axis}$$