Chap 11
DESIGN OF BEAMS & SHAFTS:

We design for:

A) Safty : $\sigma_{max} \leq \sigma_{all}$

B) Serviceability : $\delta_{max} \leq \delta_{all}$.

Design for Bending :

$$\sigma = \frac{Mc}{I} = \frac{M}{S}$$

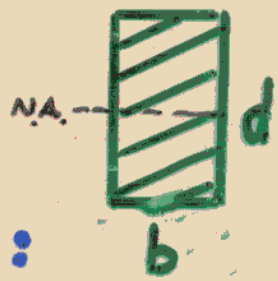
where:

$$S = \frac{I}{c} = \text{section modulus } (L^3)$$

$$\frac{M}{S} = \sigma_{all}$$

$$S_{req.} = \frac{M_{max}}{\sigma_{all}}$$ } Where M_{max} is determined from the Moment Diag.

$$S_{req.} = \frac{I}{c} = \frac{bd^3}{12 \times \frac{d}{2}} = \frac{bd^2}{6}$$



After design for Bend.:

* Check for Shear :

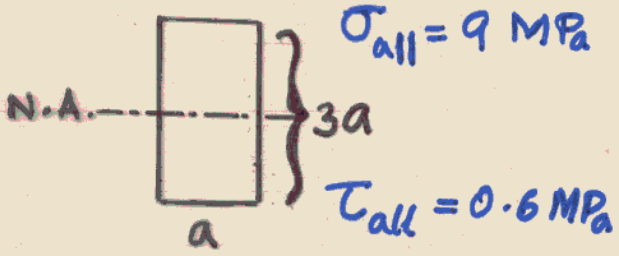
$$\tau = V_{max} Q / Ib \leq \tau_{all}$$

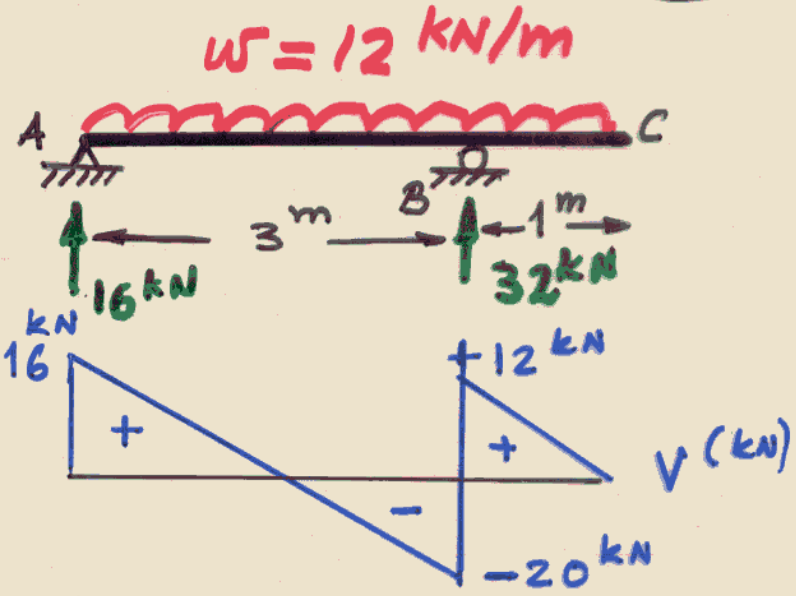
* Check for Deflection:

$$\delta \leq \delta_{all}$$

Example 1 :

Select dimension (a)?





Solution :

$$S_{req.} = \frac{M_{max}}{\sigma_{all}} = \frac{10.67}{9000} = 0.00119 \ m^3$$

$$S_{req.} = \frac{I}{c} = \frac{\frac{1}{12}(a)(3a)^3}{1.5a} = 1.5a^3 = 0.00119$$

$$a_{min} = 0.0925 \ m = 9.25 \ cm \quad (\text{say } 10 \ cm)$$

Check for Shear :

$$\tau_{max} = \frac{V_{max} Q}{I b} \leq 0.6 \ MPa$$

$$\tau_{max} = \frac{20 \times 1000 \times [0.1 \times 0.15 \times 0.075]}{\frac{1}{12}(0.1)(0.3)^3 \times 0.1} = 1 \ MPa$$

$1 \ MPa > 0.6 \ MPa$ N.G.

Select $a = 0.12 \ m$ (12 cm)

$$\tau_{max} = \frac{20000 \times 0.12 \times 0.18 \times 0.09}{\frac{1}{12}(0.12)(0.36)^3 \times 0.12} = 694444 \ Pa > 0.6 \ MPa \ (N.G)$$

Say $a = 0.13 \ m$

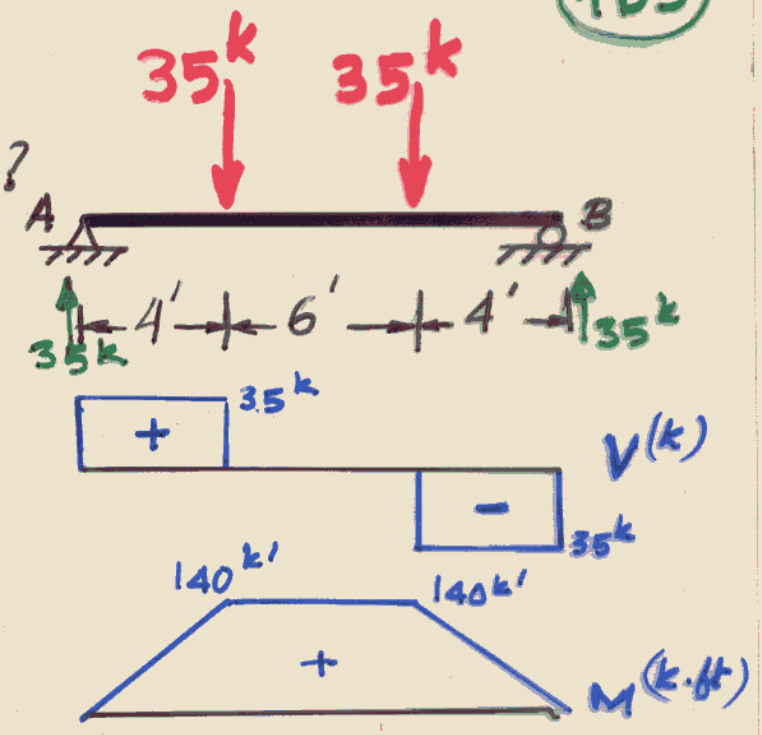
Example 2:

Given: Select Beam (AB)?

$\tau_{all} = 14.5$ ksi

$\sigma_{all} = 24$ ksi

W 18×40	$S = 68.4$ in^3
W 16×45	$S = 72.7$ in^3
W 14×43	$S = 62.7$ in^3
W 10×54	$S = 60$ in^3

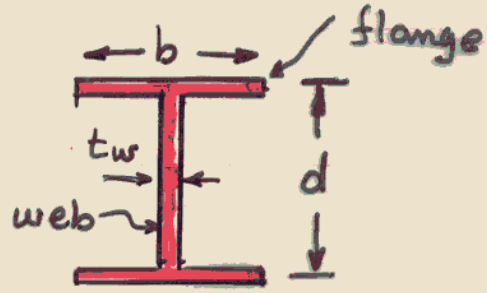


Solution:

$$S_{req.} = \frac{M_{max}}{\sigma_{all}} = \frac{140 \times 12}{24}$$

$$= 70 \text{ in}^3$$

Choose W 16 × 45



Wide Flange Beam

W 18 × 40 means

$d = 18$ in

weight/ft = 40 lb

Check shear:

Asswe web only resist shear:

$t_w = 0.25$ in

$d \approx 16$ in

$V_{max} = 35$ k

$$\tau_{ave} = \frac{V_{max}}{A_{web}} = \frac{35}{16 \times 0.25} = 8.25 \text{ ksi} \leq 14.5 \text{ ksi}$$

OK

OR

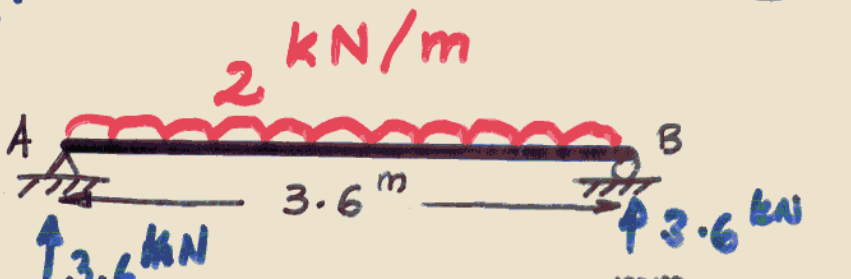
$$\tau_{max} = \frac{VQ}{Ib} = \frac{35 \times 8 \times 0.25 \times 4}{\frac{1}{12} \times 0.25 \times (16)^3 \times 0.25} = 13.125 \text{ ksi} \leq 14.5 \text{ ksi}$$

Example 3 :

Given :

$\sigma_{all} = 10$ MPa.

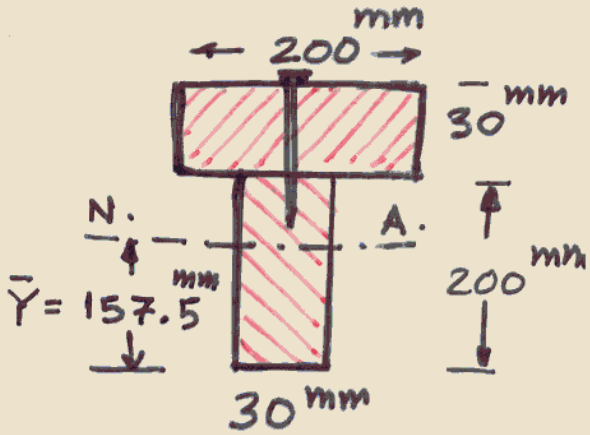
$\tau_{all} = 0.8$ MPa.



Each nail can resist 1.5 kN shear.

Required :-

- Check safty of section ?
- Provide nail spacing ?



Solution:

* From the beam loading : $M_{max} = wL^2/8 = 3.24$ kN.m

$$V_{max} = \frac{wL}{2} = 3.6 \text{ kN}$$

* From the section properties: $\bar{Y} = 157.5 \text{ mm} = 0.158 \text{ m}$

$I = 60.125 \times 10^{-6} \text{ m}^4$

* $\sigma_{max} = \frac{Mc}{I} = \frac{3240 \times 0.158}{60.125 \times 10^{-6}} = 8.5 \text{ MPa} \leq 10 \text{ MPa}$ OK

* $\tau_{max} = \frac{VQ}{Ib} = \frac{3600 \times (0.158 \times 0.03) \times \frac{0.158}{2}}{60.125 \times 10^{-6} \times 0.03} = 0.75 \text{ MPa}$

≤ 0.8 MPa OK

* $\tau b = \frac{VQ}{I} = \frac{3600 \times (0.2 \times 0.03)(0.015)}{60.125 \times 10^{-6}} = 5.4$ kN/m

* Spacing of nails :

$$S = \frac{1.5}{5.4} = 0.278 \text{ m}$$

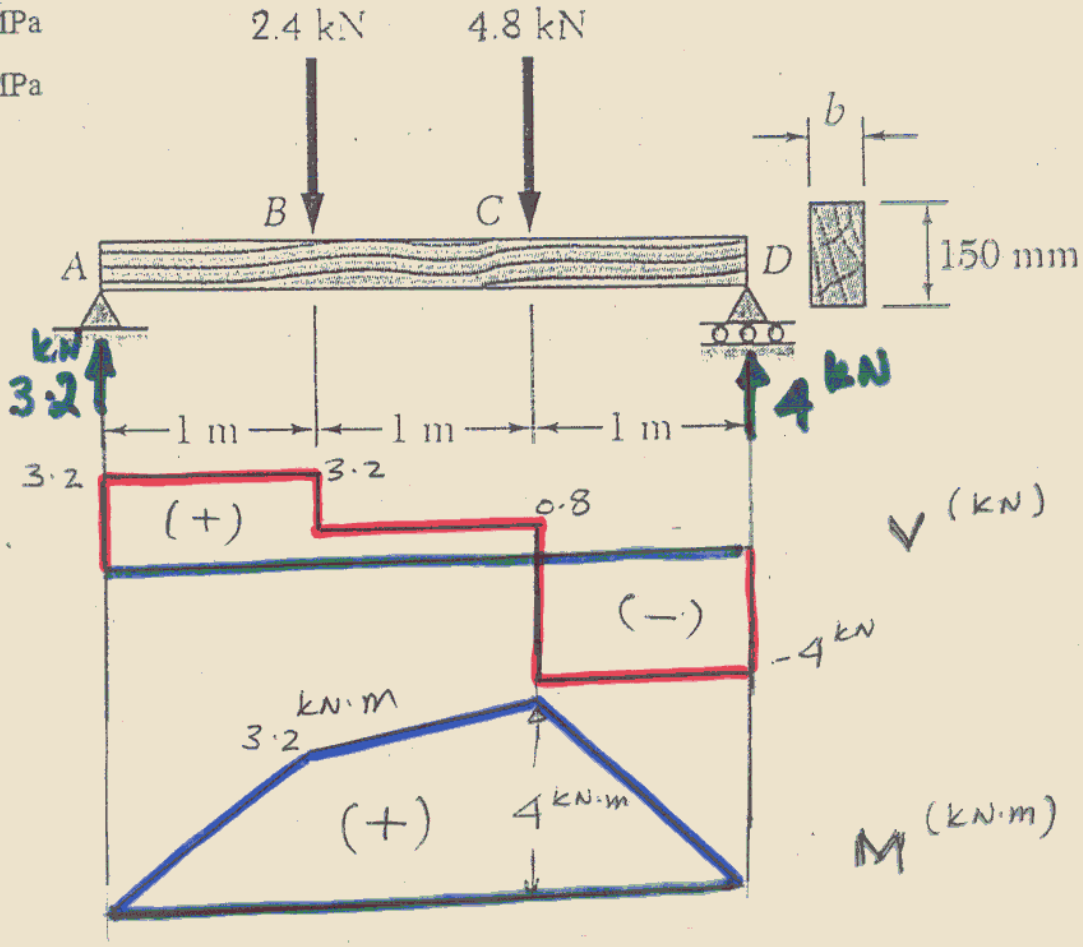
Say 25 cm spacing.

Example 4

Beam ABCD is loaded as shown below. Find minimum required width (b), given:

σ_{all} = 12 MPa

τ_{all} = 0.9 MPa



Solution:

Find Reactions:

$R_{Ay} = 3.2^{kN}$

$R_{Dy} = 4.0^{kN}$

Draw Shear & B.M. Diagrams.

$V_{max} = 4^{kN}$

$M_{max} = 4^{kN\cdot m}$

* Design for Shear:

$$\tau_{all} = \frac{VQ}{Ib} = \frac{4000\,(0.075\,b)\left(\frac{0.075}{2}\right)}{b \times (0.15)^3 \times \frac{1}{12} \times b} = \frac{40000}{b} = 9\times10^5$$

$$b = 0.044^{m}$$

* Design for Bending:

$$\sigma_{all} = \frac{Mc}{I} = \frac{4000 \times \left(\frac{0.150}{2}\right)}{\frac{1}{12} b\,(0.15)^3} = 12\times10^6$$

$$b = 0.088^{m}$$

Select $b = 100^{mm}$