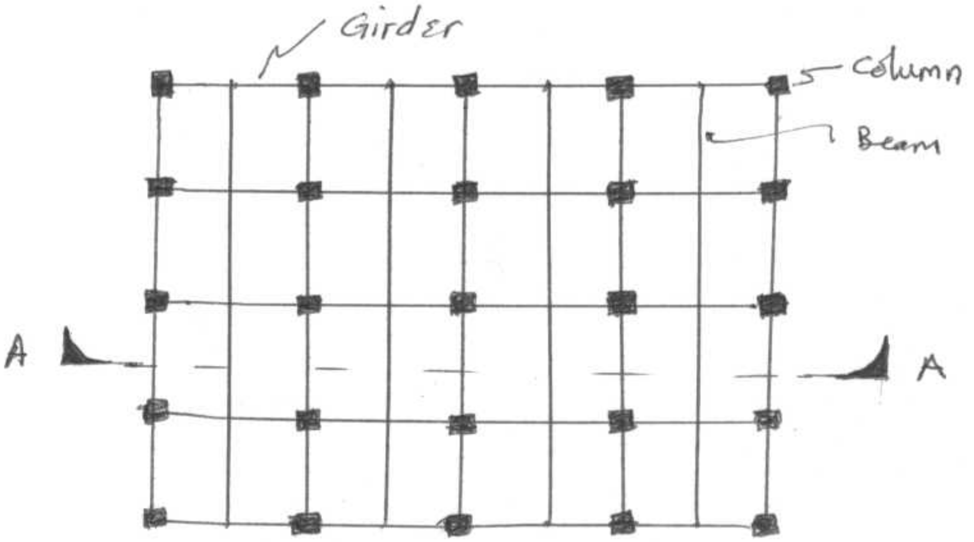
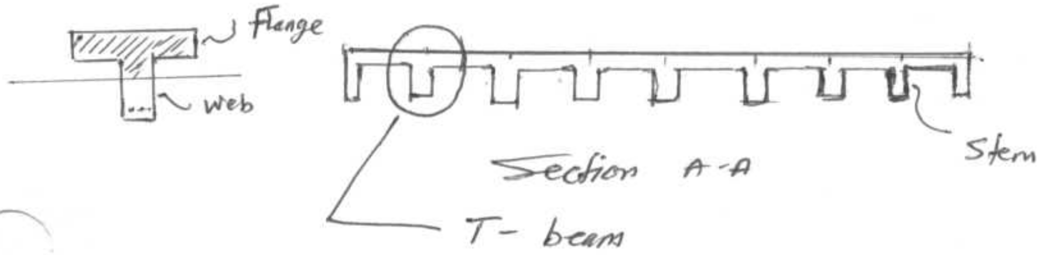
Ch. 9 T- Sections in Bending

§9.1 The T- beams are built monolithically with the slab - hence the name " T- beams

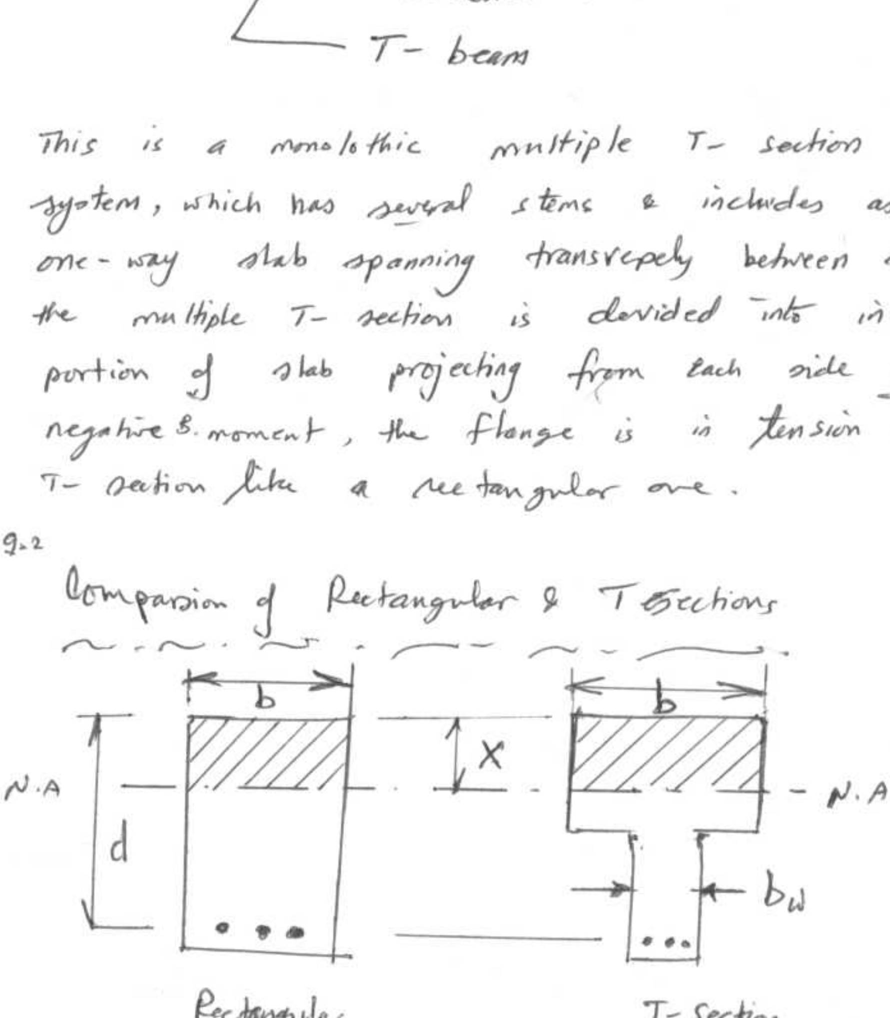




This is a monolothic multiple T- section in a slab-beam-girder system, which has several stems & includes as its flange the entire one-way slab spanning transvepely between stems. For design purpose, the multiple T- section is devided into individual T- section with portion of slab projecting from each side of stem as flanges. For negative B. moment, the flange is in tension side of the N.A. - making T- section like a rectangular one.

§ 9.2

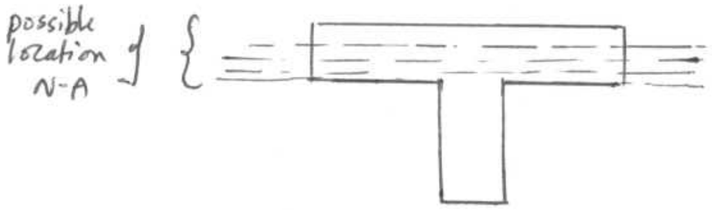
Comparsion of Rectangular & T-Sections



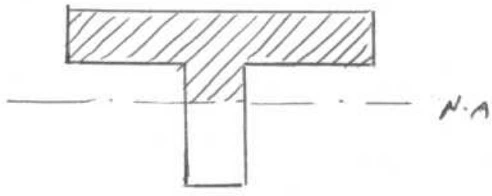
$$(M_n)_{rect.} = (M_n)_{T\text{-section}}$$

if As ~~is~~ the same is the compression area same. (equal b and x)

∴ The design and investigation of a T- section will be same as that for rectangular section provided the N.A. located within the flange. However should N.A be located in the "web", the computation of T- section will be slightly different.

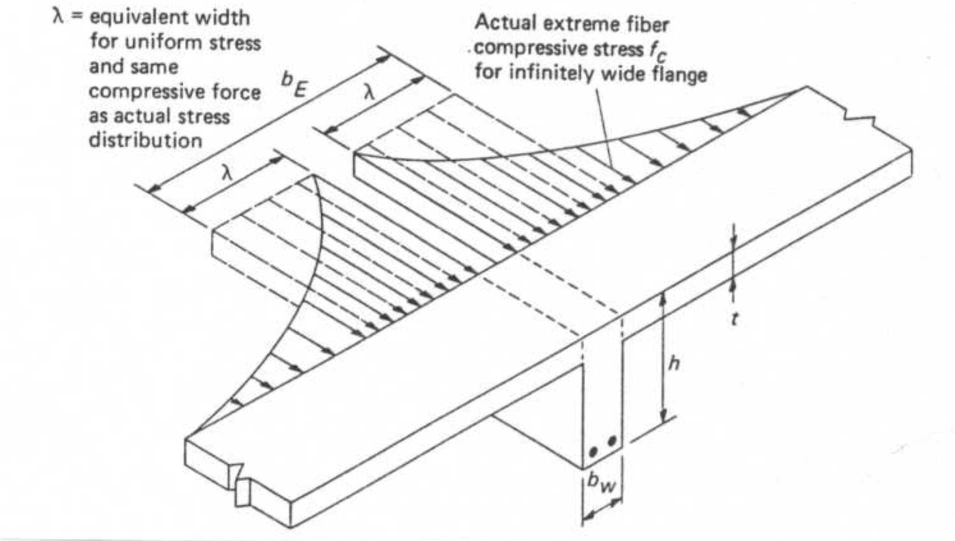


Rectangular section



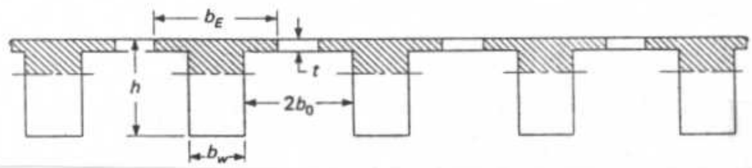
T- section

S. 9.3 Effective flange width



Effective flange width b_E over which compression stress may be considered constant. Total compressive force carried by equivalent system is same as that carried by the actual system. $\underline{\underline{b_E}}$ is noted to be function of:

* beam span length **L**
* beam spacing ℄ – ℄
* beam stem width b_w
* relative thickness of slab to total beam depth (t/h)



The ACI 10.8.2 prescribes a limit on effective flange width b_E of interior T-section to be the smallest of :

- $L/4$
- $b_w + 16t$
- ℄ - ℄ spacing of beams (℄ = center line)

The ACI 10.8.3 prescribes for exterior beams, that the b_E should be smallest of :

- $b_w + L/12$
- $b_w + 6t$
- $b_w + \frac{1}{2}$ (clear distance to next beam) $= [\, b_w + b_0 \,]$

For isolated T-beams, ACI 8.10.4 recommends

$$b_E \leq 4\, b_w$$
$$t \geq b_w/2$$

Actual b_E depends also on type of loading - Thus ACI criterion is of simplified nature.

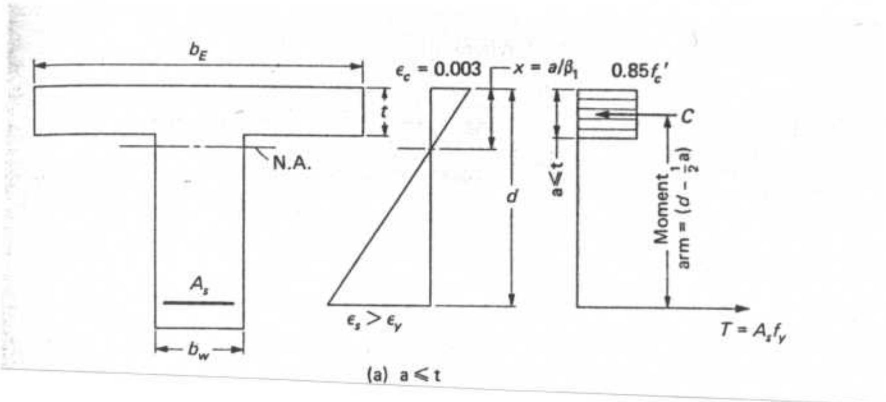
§ 9.4 Investigation of T- sections in Bending - Strength Method

Computation of nominal strength M_n is split into two categories:

- depth of compressive stress block $a \leq$ flange thickness $(a \leq t)$
- depth of compressive stress block $a > t_f$.

Case (I) $T \leq 0.85 f_c' b_E t$ $\left(A_s f_y \leq 0.85 f_c' b_E t \right) \Rightarrow a < t_f$

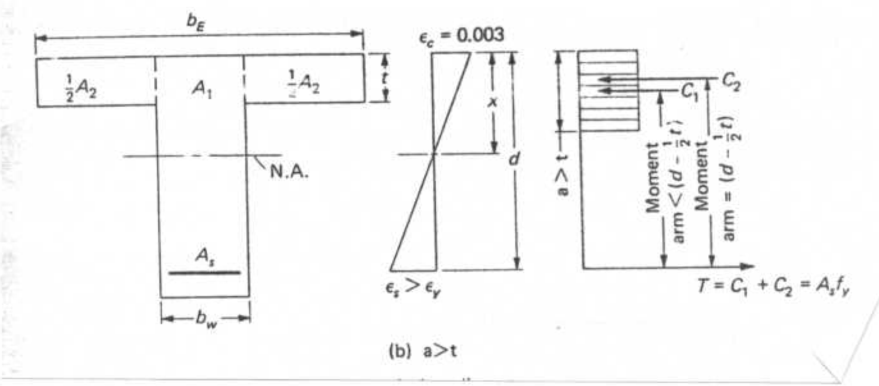
$\therefore$ treat section as rectangular.



(a) $a \leq t$

$$M_n = A_s f_y \left[d - \frac{a}{2} \right]$$

Case (II) $\left(A_s f_y > 0.85 f_c' b_E t \right)$ $a > t_f$



(b) $a > t$

$$M_n = C_1 \left(d - \frac{a}{2}\right) + C_2 \left(d - \frac{t}{2}\right)$$

$$C_1 = .85 f_c' b_w a$$
$$C_2 = .85 f_c' (b_E - b_w) t$$

$$T = C_1 + C_2 \Rightarrow a$$

$$a = \frac{A_s f_y - C_2}{.85 f_c' b_w}$$

Ex. 9.4-1

Determine nominal moment strength M_n within the span of a floor beam whose projection below a $4\frac{1}{2}''$ slab is 13×24 in ($d = 25''$ for 2 layers). $A_s = 8\#8$. Beam span is $26'$ & beams spaced at $13'$. $f_c' = 3000$ psi, $f_y = 50{,}000$ psi

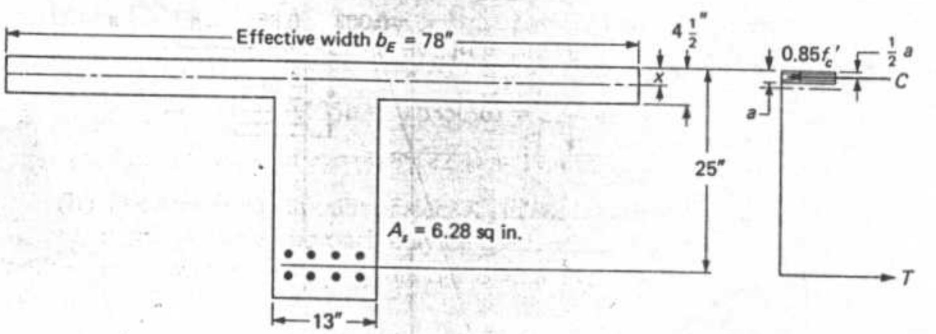


Figure 9.4.2 T-section for Example 9.4.1.

Solution.

Find b_E according to ACI 8.10.2 is the smallest of

1) $\frac{L}{4} = \frac{26(12)}{4} = 78''$ ⟸ Control.
2) $b_w + 16t = 13 + 16(4.5) = 85''$
3) ℄ - ℄ $= 13'(12) = 156''$

$\therefore \quad b_E = 78''$

Find limiting of A_s ∀ $a = 4.5$ in

$$A_s = \frac{0.85 f_c' \, b_E \, t}{f_y} = \frac{0.85(3)\,78\,(4.5)}{50} = 17.9 \text{ in}^2$$

Since A_s provide $(8\#8) = 6.28$ in < 17.9 in

$\therefore \quad a < t$

$\therefore$ treat it as rectangular section.

$$M_n = A_s f_y \left[d - \frac{a}{2}\right]$$

$$= (50)(6.28)\left[25 - \frac{1.58}{2}\right]\left(\frac{1}{12}\right)$$

$$= 633 \text{ ft-k}$$

$$a = \frac{f_y A_s}{0.85 f_c' b} = \frac{(50)(6.28)}{(0.85)(3)(78)}$$

$$a = 1.58$$

Ex. 9.4.2

Determine the nominal moment strength M_n of the isolated T- section shown with $A_s = 8 \# 11$ $(12.48) in^2$. $f_c' = 3$ ksi , $f_y = 50$ ksi

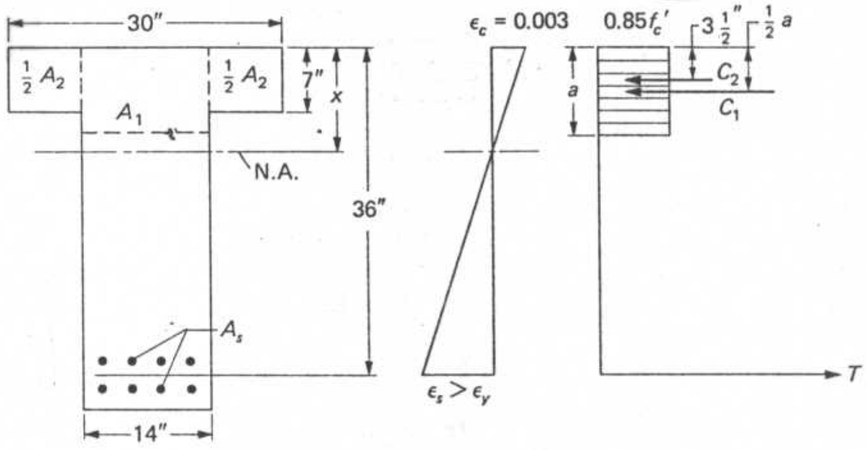


Figure 9.4.3 T-section for Examples 9.4.2, 9.5.1, and 9.6.1.

$$\frac{\varepsilon_s}{36 - x} = \frac{0.003}{x}$$

$x = 11.15$

$\varepsilon_s = 0.0067 >$

$\varepsilon_y = 0.001724$

Solution

Check ACI 8.10.4 (isolated T-section)

$b_E \not> 4 b_w \not> 4(14) = 56''$ (ok ∴ $b_E = 30''$)

$t \leq \frac{b_w}{2} = \frac{14}{2} = 7''$ ok

Find limiting A_s ∀ $a = t$.

$$A_s = \frac{.85 f_c' b_E t}{f_y} = \frac{(.85)(3)(30)(7)}{50} = 10.70 \text{ in}$$

which is smaller than that provided $A_s = 12.48$

∴ $a > t$

∴ treat section as T- section

$T = A_s f_y = (12.48)(50) = 624$ k

$C = C_1 + C_2 = 0.85 f_c' b_w a + (.85) 3 (b_E - b_w) t$

$C = C = 0.85(3)(14)(a) + (.85)(3)(30 - 14) 7$

$T = C \Rightarrow a = 9.48''$ $\quad x = \frac{a}{\beta_1} = 11.15''$

$C_1 = 338.4$ k

$C_2 = 285.6$ k

$a = 9.48''$

$t = 7''$

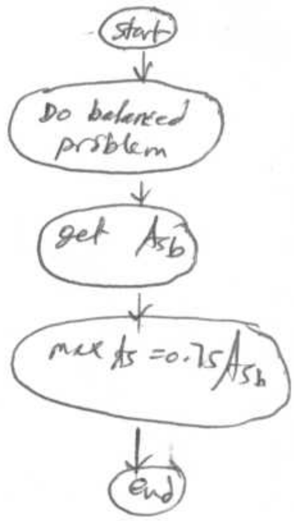
∴ $\varepsilon_s > \varepsilon_y$ and steel has yielded

$$M_n = C_1 [36 - \frac{a}{2}] + C_2 [36 - \frac{t}{2}] = \underline{1,656 \text{ k-ft}}$$

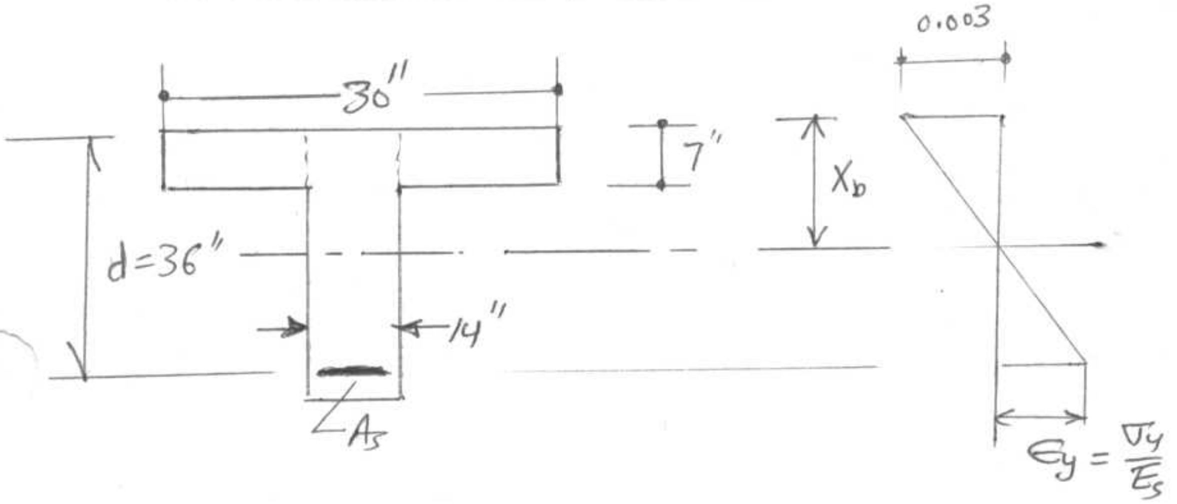
Maximum Tension Reinforcement Permitted in T-Sections

For T-section of the figure below. Determine max. reinforcement ratio ρ & area A_s permitted by code. Use $f_c' = 3000$ psi & $f_y = 50{,}000$ psi

Note, for rectangular sections, $x = 0.75\, x_b$ is not same as $A_s = 0.75\, A_{sb}$, ∵ b is varying.
∴ Use $A_s^{max} = 0.75\, A_{sb}$ for non-rectangular sections



Do balanced problem first.



$$x_b = \frac{0.003}{0.003 + f_y/E_s}\, d = \frac{0.003}{0.003 + 50/29\times10^3}(36) = 22.9''$$

$$a_b = \beta_1 x_b = 0.85\,(22.9) = 19.4 \text{ in} \quad (> t)$$

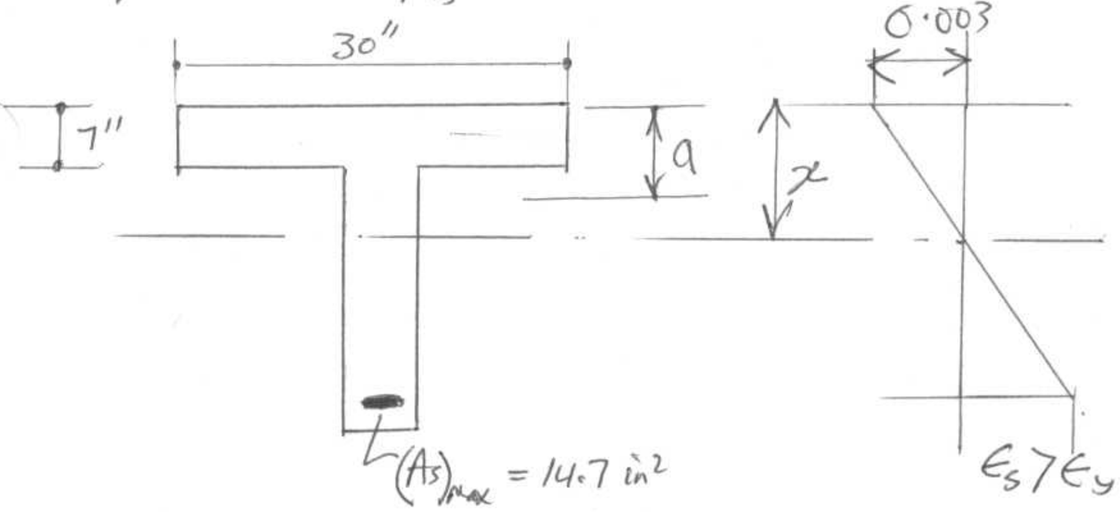
$$C_{1b} = 0.85 f_c'\, b_w\, a_b = 0.85 * 3 * 14 * 19.4 = 693 \text{ kips}$$
$$C_{2b} = 0.85\, f_c'\,(b_E - b_w)t = 0.85 \cdot 3\,(30-14)\,7 = 286 \text{ kips}$$
$$T_b = A_{sb}\,(50)$$

$$T_b = C_{1b} + C_{2b} \Rightarrow A_{sb} = 19.6 \text{ in}^2$$

ACI 10.3.3 $\quad \text{max } A_s = 0.75\, A_{sb} \quad$ max $A_s = 14.7$ in^2

$$\rightarrow \max \rho = \frac{\max A_s}{b_E d} = \frac{14.7}{(30)(36)} = 0.0136$$

Find the location of N.A. which correspond to max A_s.



Assume $a > t$

$$C_1 = (0.85)(3)(14)\,a = 35.7\,a$$
$$C_2 = (0.85)(3)(30-14)\,7 = 286\ k$$
$$T = (14.7)(50) = 735\ k$$

$$T = C_1 + C_2 \implies a = \frac{735-286}{35.7} = 12.6'' > t \ \text{ok}$$

$$x = \frac{a}{\beta_1} = \frac{12.6}{0.85} = 14.8''$$

$$\Rightarrow \frac{x}{x_b} = \frac{14.8}{22.9} = 0.65$$

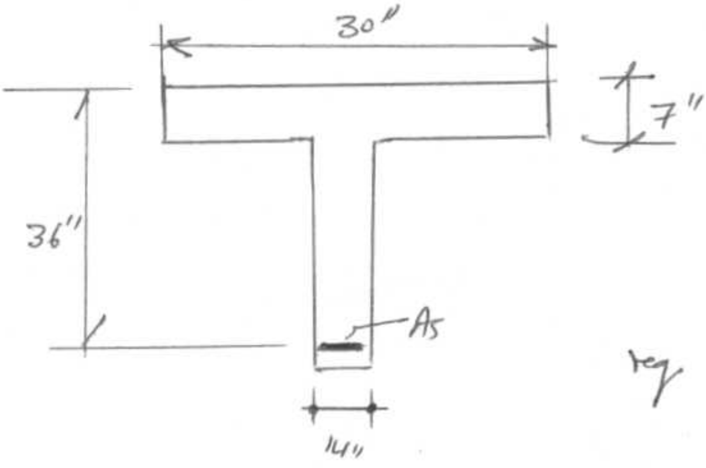
$\therefore$ Although $\left(A_s^{max} = 0.75\,A_{sb}\right)$, $x = 0.65\,x_b$

However for rectangular section if $A_s^{max} = 0.75\,A_{sb}$ then $x = 0.75\,x_b$.

$\therefore$ More ductility is required for a T-section than a single or / double reinforced sections.

Ex. 9.7.1
P340

Determine $A_s = ?$ in T-section of Fig. 9.4.3 for $M_{DL} = 370$ ft-k & $M_{LL} = 520$ ft-k. Use $f'_c = 3000$ psi, $f_y = 50,000$ psi



$$M_u = 1.4 M_{DL} + 1.7 M_{LL} = 1,402 \text{ ft-k}$$

$$\text{req. } M_n = \frac{M_u}{\phi} = \frac{1402}{0.9} = 1560 \text{ ft-k}$$

Find the moment capacity such that only flange in comp. $a = t$.

$$C = 0.85 f'_c b_E t = .85(3)(30)(7) = 536 \text{ k}$$

$$M_n = C\left(d - \frac{a}{2}\right) = 536\left(36 - \frac{7}{2}\right)\frac{1}{12} = 1450 \text{ ft-k} < \text{req. } M_n$$

∴ We need more compression ⇒ $a > t$

∴ Use two-Couple Method.

$$M_n = 0.85 f'_c b_w a\left(d - \frac{a}{2}\right) + 0.85 f'_c (b_E - b_w) t \left[d - \frac{t}{2}\right]$$

$$(1560)(12) = 0.85(3)(14)\, a \left(36 - \frac{a}{2}\right) + 0.85(3)(30 - 14)7(36 - 7/2)$$

$$a^2 - 72a + 527 = 0 \quad \Rightarrow \quad a = 8.3''$$

$$\therefore\ C = C_1 + C_2 = .85(3)(14)(8.3) + .85(3)(30 - 14)(7) = 582^k$$

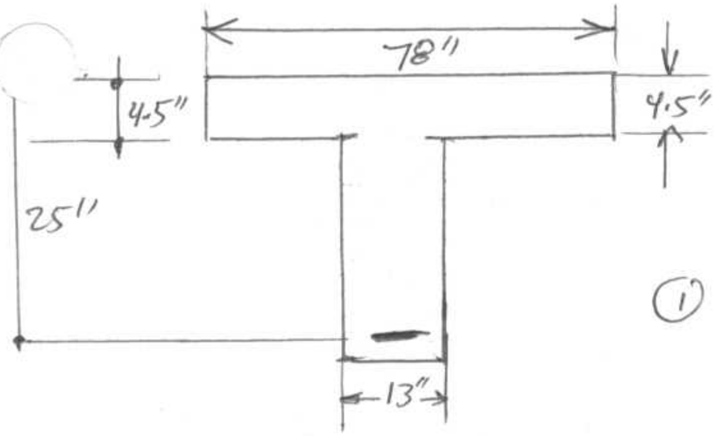
$$C = T \Rightarrow 582 = 50 A_s \quad \Rightarrow \quad A_s = \frac{582}{50} = 11.64 \text{ in}^2$$

$< A_s^{max}$ (steel will yield)

∴ Use 8 #11.

Ex. 9.7.2
P. 341

Design steel reinforcement for section of Fig 9.4.2 to carry a factored moment M_u of 735 ft-k
Use $f'_c = 3$ ksi & $f_y = 50$ ksi



① Calculate the moment capacity if $a = t$.

$$C = .85 f'_c b_E t = (0.85)(3)(78)(4.5) = 895^k$$
$$M_n = C\left(d - \frac{a}{2}\right) = 895\left(25 - \frac{4.5}{2}\right)\frac{1}{12} = 1597 \text{ ft-k}$$

which is larger than what is needed.

$\therefore$ $a < t$ ($\therefore$ treat as rectangular section)

① approach I: Calculate $R_n = \dfrac{M_u}{\Phi b_E d^2}$ then find ρ & A_s

② Approach II: By trail & error.

Try $\text{arm} = \left(d - \frac{a}{2}\right) = 0.9d$

$$A_s = \frac{M_n}{f_y\left(d - \frac{a}{2}\right)} = \frac{817\,(12)}{(50)(0.9)(25)} = 8.71 \text{ in}^2$$

$\therefore$ try $4\#9$ & $4\#10$ $\Rightarrow$ $A_s = 9.08 \text{ in}^2$

Check:

$$C = T \quad \left[(0.85)(3)(78)\,a = (9.08)\,50\right] \Rightarrow a = 2.28$$
$$\text{arm} = 25 - \frac{2.28}{2} = 23.86$$
$$A_s = \frac{M_n}{f_y\left(d - \frac{a}{2}\right)} = \frac{(817)(12)}{(50)(23.86)} = 8.22 \text{ in}^2$$

$\therefore$ try $4\#8$ & $4\#10$ $\Rightarrow$ $A_s = 8.24 \text{ in}^2$

$$C = T \quad \left[198.9\,a = 50\,(8.24)\right] \Rightarrow a = 2.07 \text{ in}^2$$
$$\text{arm} = \left(25 - \frac{2.07}{2}\right) = 23.96''$$
$$M_n = T\left(d - \frac{a}{2}\right) = 412\,(23.96)\left(\frac{1}{12}\right) = 823 \text{ ft-k}$$
$$\Phi M_n = (0.9)(823) = 740 \text{ ft-k} > M_u = 735 \text{ ft-k}$$