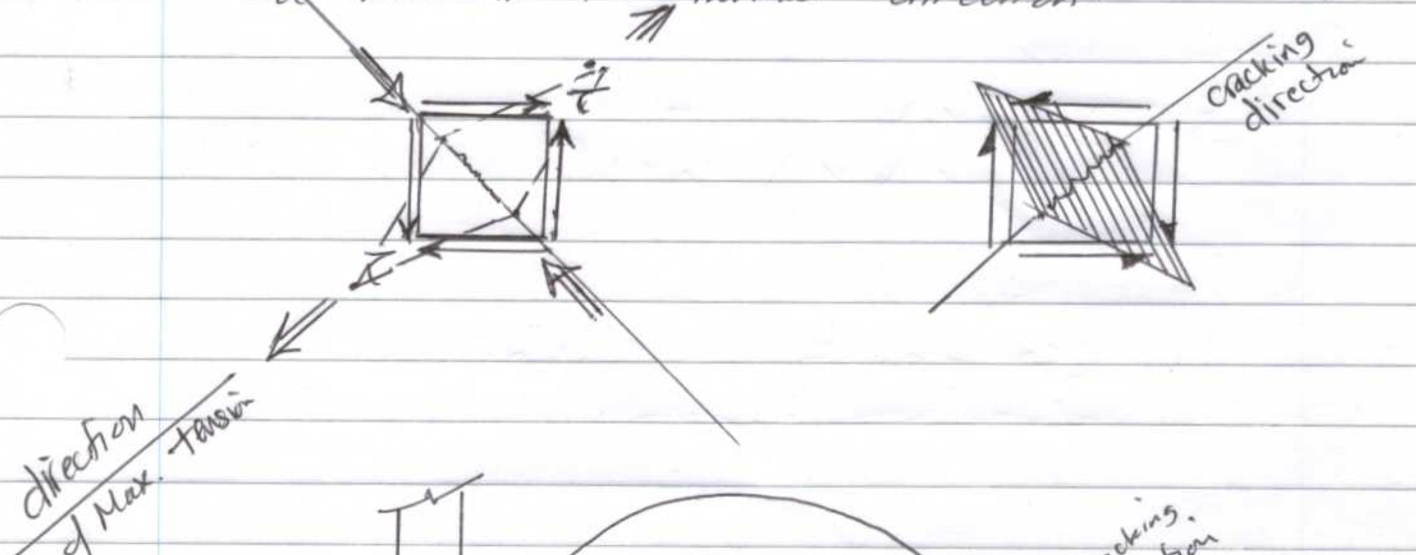
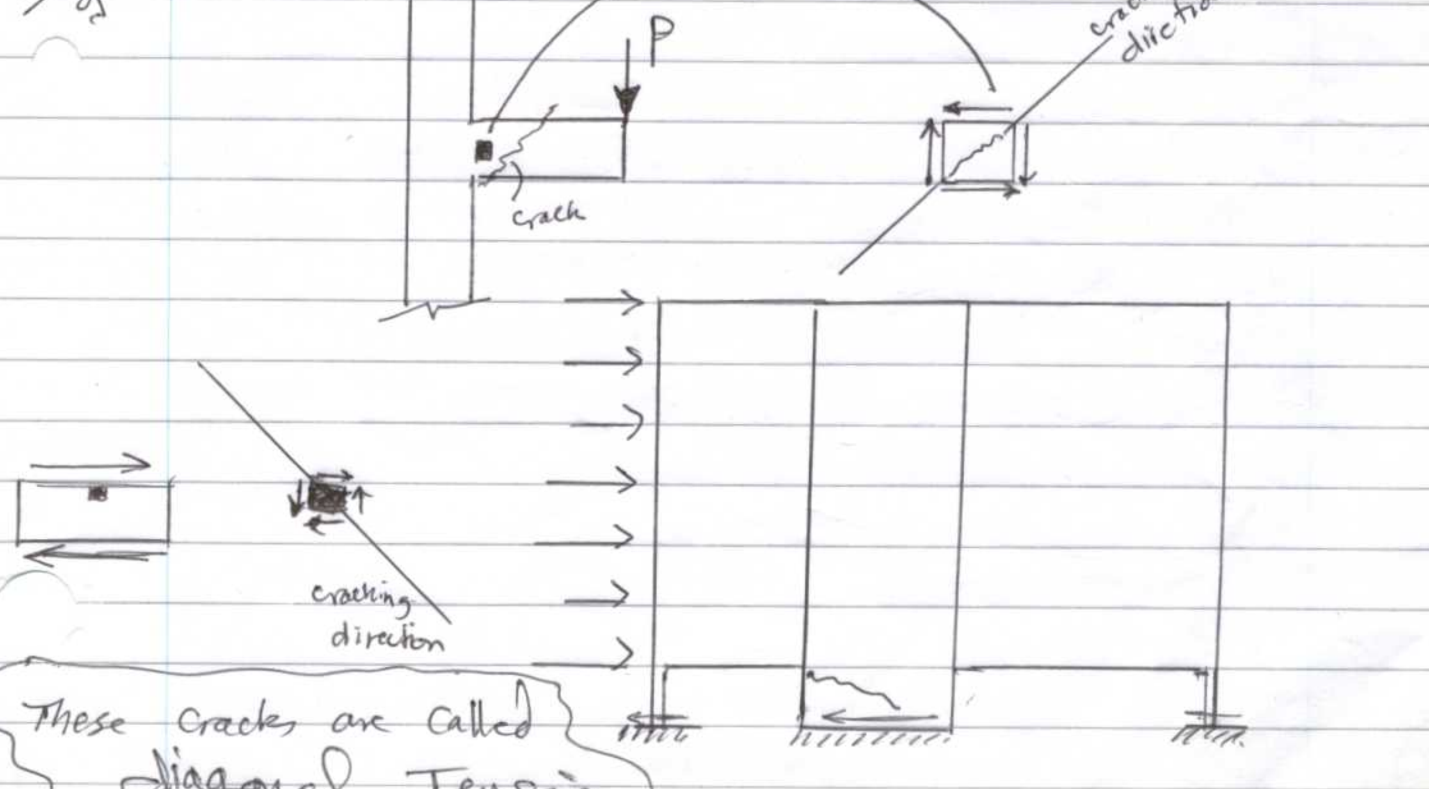
Shear in Reinforced Concrete Beams

shear stress is caused by shear force and normally maximum near support. From elemental equilibrium, shear stress must be present in all four faces. For brittle material like concrete, the state of pure shear will generate maximum tensile force in a 45° inclination along which concrete will fail by developing a crack in the normal direction.



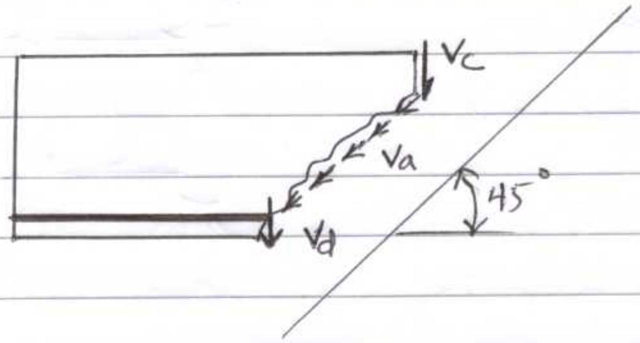


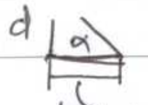
These cracks are called Diagonal Tension

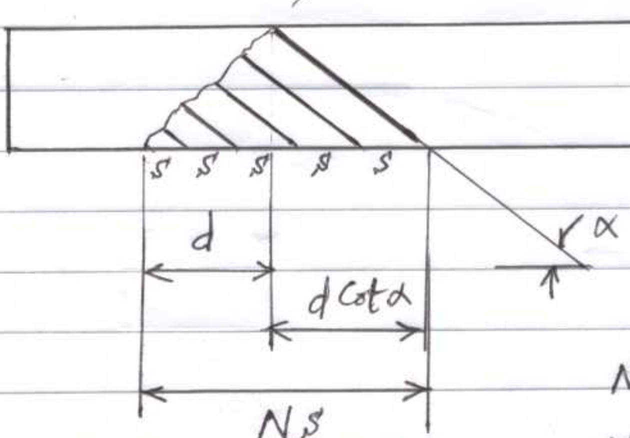
The trasfer of shear in reinforced concrete members occurs by combination of the following mechanism:

1) shear resistance of the uncracked concrete (V_c)
2) Aggregate interlock, tangent along crack (V_a)
3) Dowel action resistance of the longitudinal reinforcement. (V_d)

* 4) shear reinforcement from stirrups (V_s)

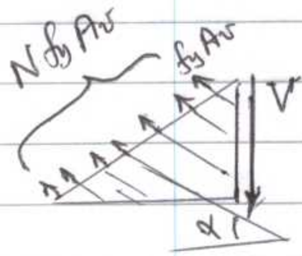






S = spacing of stirrups
N = # of stirrups to close inclined crack

α (inclination of stirrups)



$$NS = d + d\cot\alpha$$

$$N = \frac{d(1+\cot\alpha)}{S}$$

$\Sigma F_y = 0$ $\quad V = V_s = N f_y A_v \sin\alpha$ (total shear force carried by stirrups)

$$V_s = \frac{d(1+\cot\alpha)}{S} \sin\alpha \, f_y A_v \Rightarrow V_s = \frac{A_v f_y (\sin\alpha + \cos\alpha)}{S} d$$

General.

For virtical stirrups $\alpha = 90 \Rightarrow$

$$V_s = \frac{A_v f_y d}{S}$$

shear force carried by virtical stirrups.

Design for shear

The major portion of shear force will be carried by concrete (uncracked) and stirrups. such that the factored ultimate shear V_u will be equal to :

$$\boxed{V_u = \phi\,(V_s + V_c)}$$

V_s = shear force carried by stirrups
V_c = shear force carried by concrete
ϕ = 0.85 (understrength factor

$$\boxed{\begin{aligned} V_c &= 2\sqrt{f_c'}\; b\,d \\ V_s &= \frac{A_v f_y d}{s} \end{aligned}}$$

s = spacing of stirrups.
A_v = cross section area of stirrups (#3 or #4)

The maximum spacing of stirrups is the smaller of ($d/2$ or $A_v f_y / 50b$)

In shear design one of the two things will be calculated

① s is known and it is required to find V_u and location

$$\boxed{s \longrightarrow v_s \longrightarrow V_u = \phi\,(v_s + 2\sqrt{f_c'}\,bd)}$$

② V_u is given and it is required to find s

$$\boxed{V_u \rightarrow v_s = \frac{V_u}{\phi} - 2\sqrt{f_c'}\,bd \rightarrow s = \frac{A_v f_y d}{v_s}}$$

<u>Remark</u>: For the region near support, design shear using shear force not at the support but use a reduced force which is at a distance d frome face of support.

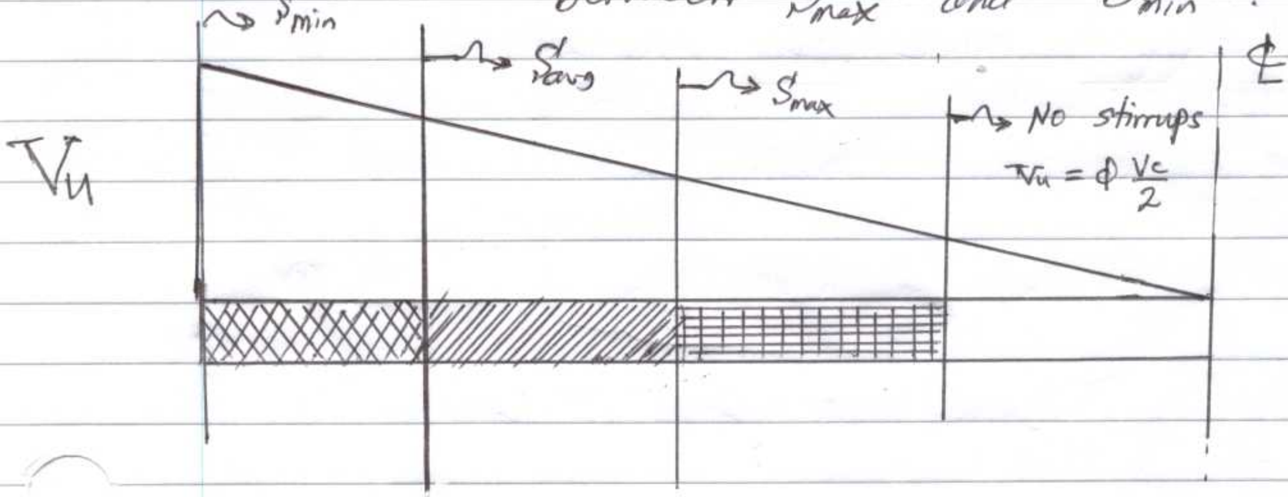
The span of the beam will be divided into four categories:

A) No stirrups is required and shear force is carried by concrete only

(B) The region of max. spacing, but need to determine it starts.

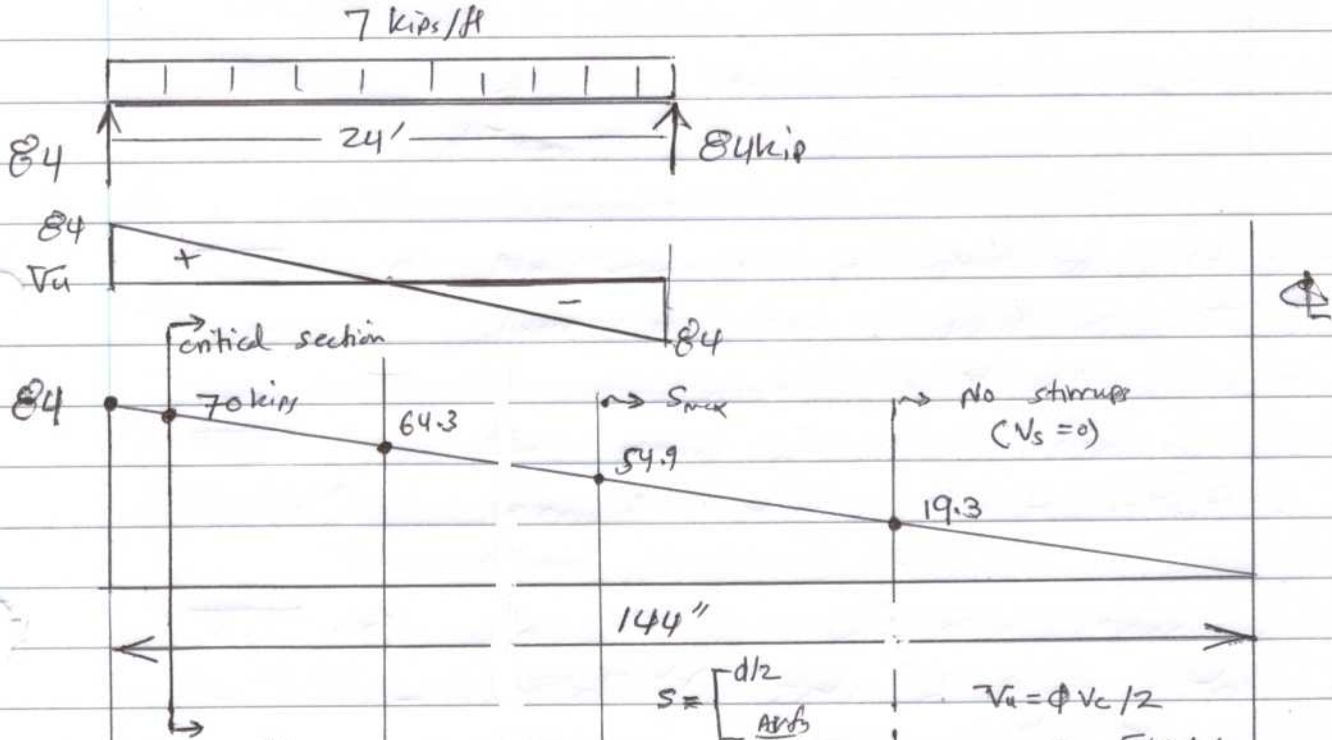
(C) The region of min spacing which is at the zone of high shear force near support.

d) The intermidiate region which is between S_{max} and S_{min}.



Finding S and the range $\Longrightarrow$ shear design

Ex. 8.1 : A simply supported beam of 24 ft-long must carry a uniform load $w_u = 7$ kips/ft which includes its own self-weight. Design the required shear reinforcement (stirrups). Given $f'_c = 3500$ psi, $f_y = 40$ ksi, $b = 16$ in & $d = 24$ in



$\frac{V_{crit}}{144-d} = \frac{84}{144}$

$V_{critical} = 70$ kips

$V_u = 70$

$V_c = 2\sqrt{f'_c}\,bd = 45.4$

$V_s = \frac{70}{.85} - 45.4$

$V_s = 37$ kips

$S' = \frac{A_v f_y d}{V_s}$

$= 5.7$ in

let $S = 5$

$S = 5$

let $S = 7$

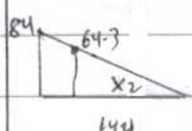
$V_s = \frac{A_v f_y d}{S'}$

$= 30.2$

$V_c = 45.4$

$V_u = \phi(V_s + V_c)$

$V_u = 64.3$



$\frac{x_2}{64.3} = \frac{144}{84}$

$x_2 = 110$

$S = \begin{cases} d/2 \\ \frac{A_v f_y}{50b} \end{cases}$

smallest $= \left\langle \begin{matrix} 12 \\ 11 \end{matrix} \right.$

$\therefore S_{max} = 11$ in

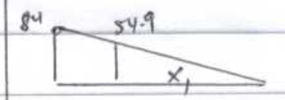
$V_s = \frac{A_v f_y d}{S}$

$= \frac{(.22)(40)(24)}{11}$

$= 19.2$

$V_u = 0.85(19.2 + 2\sqrt{f'_c}\,bd)$

$V_u = 54.9$ kips



$\frac{x_1}{54.9} = \frac{144}{84}$

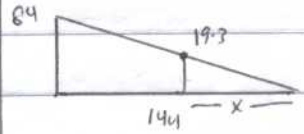
$x_1 = 94.1$

$S = 11$

$V_u = \phi V_c / 2$

$= \phi\, 2\sqrt{f'_c}\,bd/2$

$= 19.3$ kips

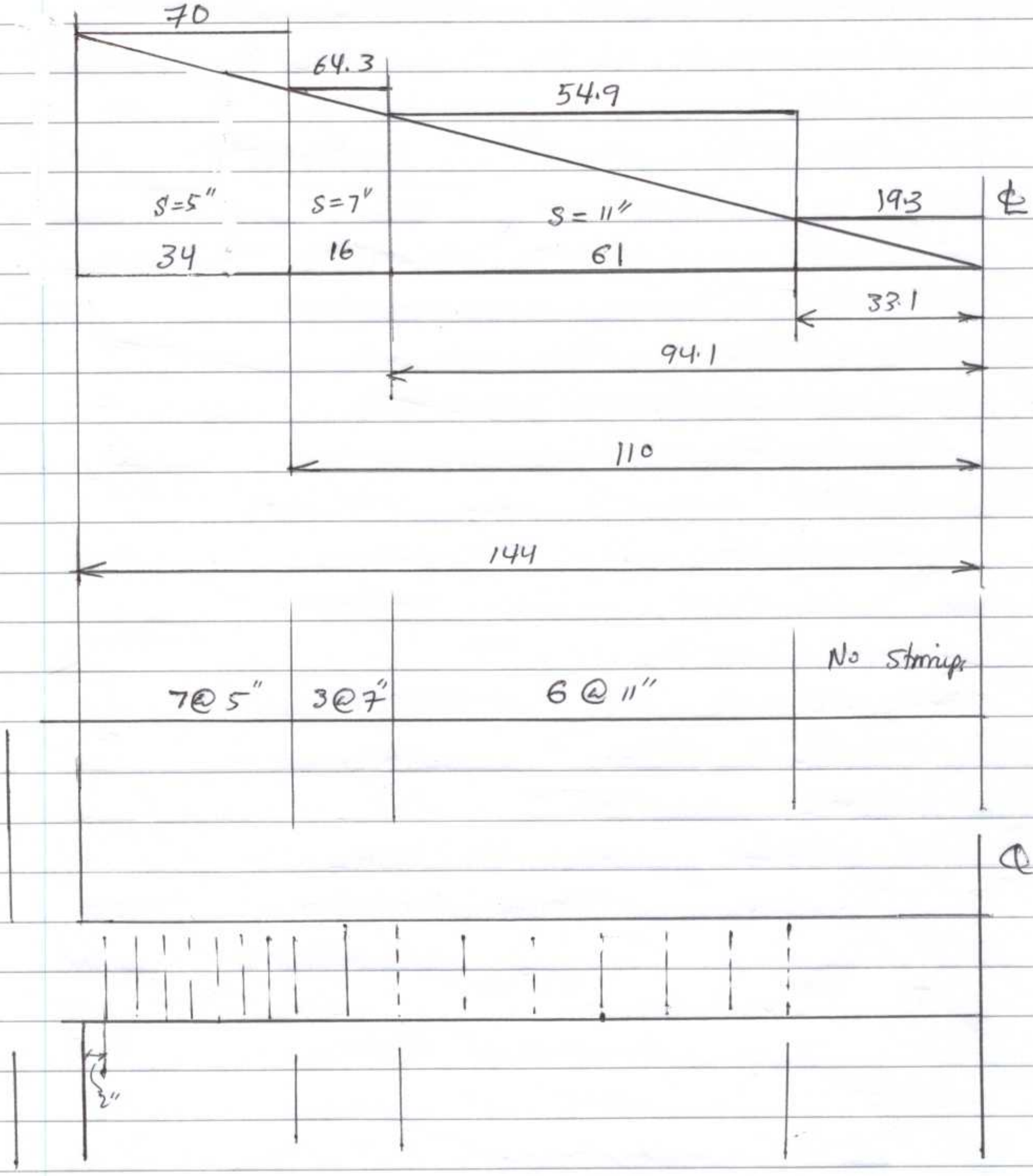


$\frac{84}{144} = \frac{19.3}{x}$

$x = 33.1$

$S' = \infty$

SUMMARY


70
64.3
54.9
s=5"
s=7"
s = 11"
19.3
℄
34
16
61
33.1
94.1
110
144
No stirrup
7@5"
3@7"
6@11"
℄
2"