Ch. 6 Development of Reinforcement

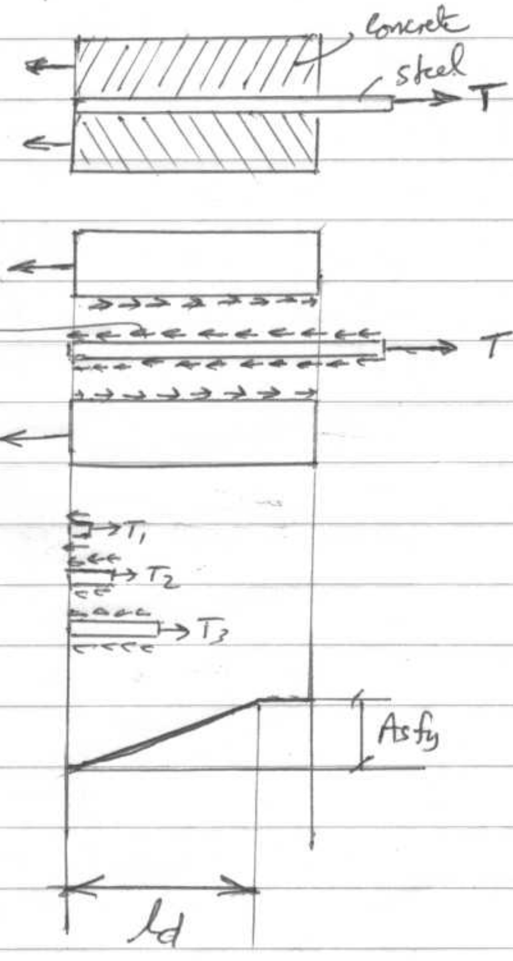
Behavior of the Reinforced concrete (RC) members:

- Bending (concrete + steel)
- Shear (concrete + stirrups)
- Bond (development of reinforcement)

Two types of failure under bond

① Pullout failure

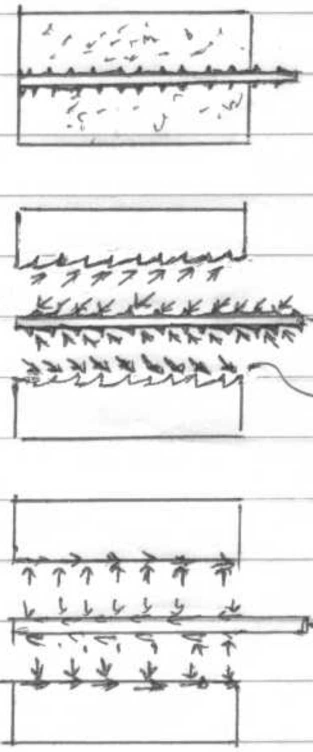
plain bars



Force in Steel developed gradually

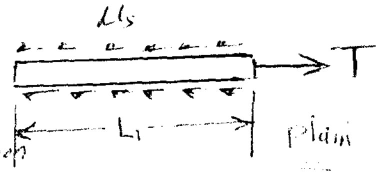
② Splitting failure

deformed bars with lugs



$L_1 = AB$. If u_s is the failure stress against slippage acting over the nominal surface area $\pi d_b L_1$, then

$$\underbrace{u_s \pi d_b L_1}_{\text{surface Area}} = f_y \underbrace{\pi \frac{d_b^2}{4}}_{\text{cross section area}}$$

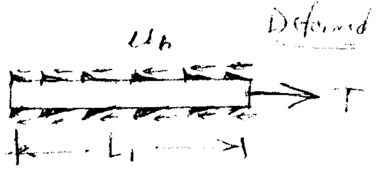


or

$$\boxed{L_1 = \frac{f_y}{4u_s} d_b} \tag{6.2.1}$$

On the other hand, if u_b is the failure stress against splitting and A_{br} is the average bearing area per unit length, then

$$u_b A_{br} L_1 = f_y \pi \frac{d_b^2}{4}$$



or

$$\boxed{L_1 = \frac{f_y}{A_{br} u_b} \pi \frac{d_b^2}{4}} \tag{6.2.2}$$

The same situation exists in free body BC, as shown in Fig. 6.2.1(c). Thus the maximum tensile force at B has to develop by embedment in both directions from B; that is, both the AB and BC distances. Where space limitations prevent providing the proper amount of straight embedment, such bars may be terminated by standard hooks (as defined in ACI-7.1). A standard hook is permitted to be considered as contributing to an equivalent development length by mechanical action (ACI-12.5), thus reducing the total embedment dimension required. Section 6.11 provides treatment of development length with standard hooks.

Adequate development length must be provided for a reinforcing bar in compression as well as in tension.

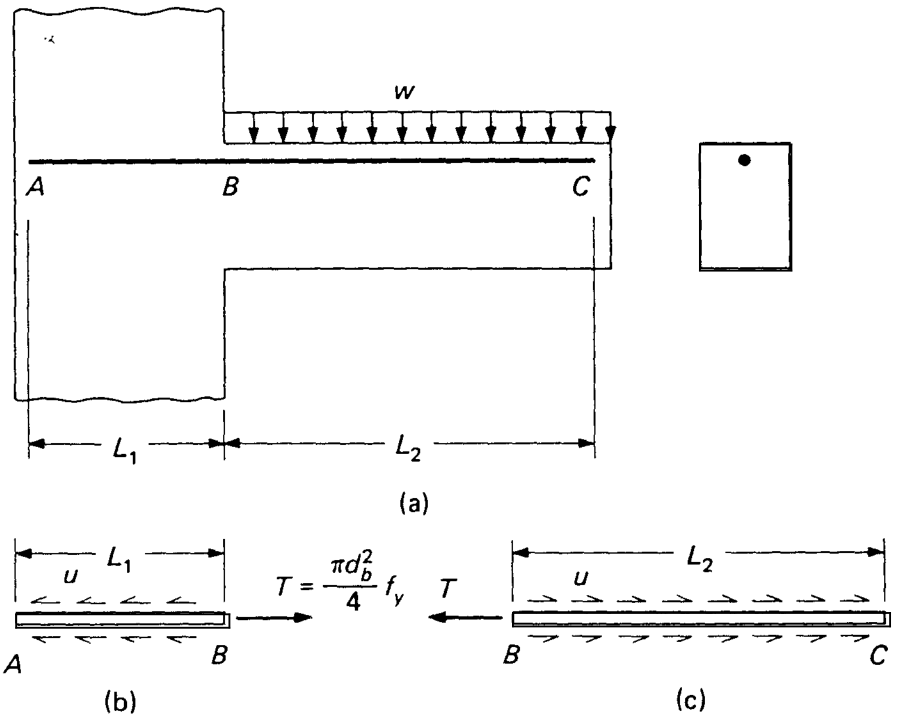


Figure 6.2.1 Development of reinforcement.

6.4 Failure Modes

The term "bond failure" has been given to the mechanism by which failure occurs when inadequate development length is provided. Years ago, when plain bars (relatively smooth bars without lug deformations) were used, slip resistance ("bond") was thought of as adhesion between concrete paste and the surface of the bar. Yet even with low tensile stress in the reinforcement, there was sufficient slip immediately adjacent to a flexural crack in the concrete to break the adhesion, leaving only friction to resist bar movement relative to the surrounding concrete over the slip length.

Shrinkage can also cause frictional drag against the bars. Typically, a hot-rolled *plain* bar may pull loose by longitudinal splitting when the adhesion and friction resistances are high, or just pull out leaving a cylindrical hole when adhesion and friction resistances are low.

Deformed bars were created to change the behavior pattern so that there would be less reliance on friction and adhesion (though they still exist) and more reliance on the bearing of the lugs against the concrete. The bearing forces act at an angle to the axis of the bar, causing radial outward components against the concrete, as shown in Fig. 6.4.1. When inadequate development length is provided, deformed bars in normal-weight concrete give rise to a splitting mode of failure (i.e., "bond failure") [6.1, 6.5, 6.7]. A splitting failure occurs when the wedging action of the steel lugs on a deformed bar causes cracks in the surrounding concrete parallel to the bar. These cracks occur between the bar and the nearest concrete face, as shown in Fig. 6.4.2(a, b), or over the short distance between bars when bars are closely spaced, as in Fig. 6.4.2(c).

When small size bars are used with large cover, the lugs may crush the concrete by bearing and result in a pullout failure without splitting the concrete. This nonsplitting failure has also been reported for larger bars in structural lightweight concrete [6.1].

Although splitting is the usual failure mode, an initial splitting crack on one face of a beam is *not* considered failure. The distress sign indicating failure is *progressive splitting*. Confinement of tension steel by stirrups, ties, or spirals usually will delay collapse (commonly defined as an increase in loading that results in no increase in resistance) until several splitting cracks have formed.

Originally, development length requirements were based on pullout tests [6.8] of plain bars, followed by pullout tests [6.9–6.15] of deformed bars, including the related load-slip data. Since confinement exists in pullout tests, the early work did not give sufficient emphasis to the splitting mode of failure. Splitting has been emphasized in the more recent studies by Orangun, Jirsa, and Breen [6.3, 6.4],

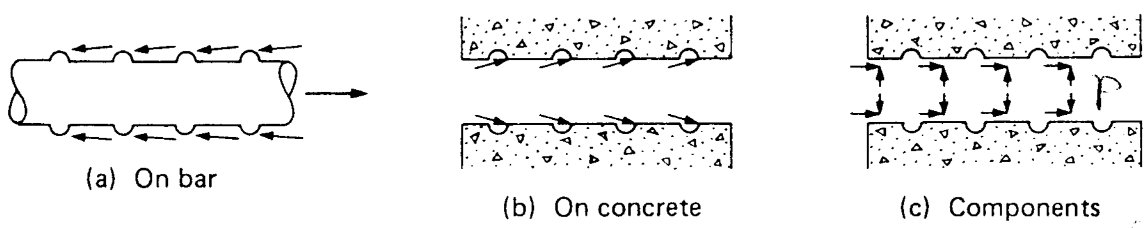


Figure 6.4.1 Forces between bar and surrounding concrete.

concrete cylinder under pressure P

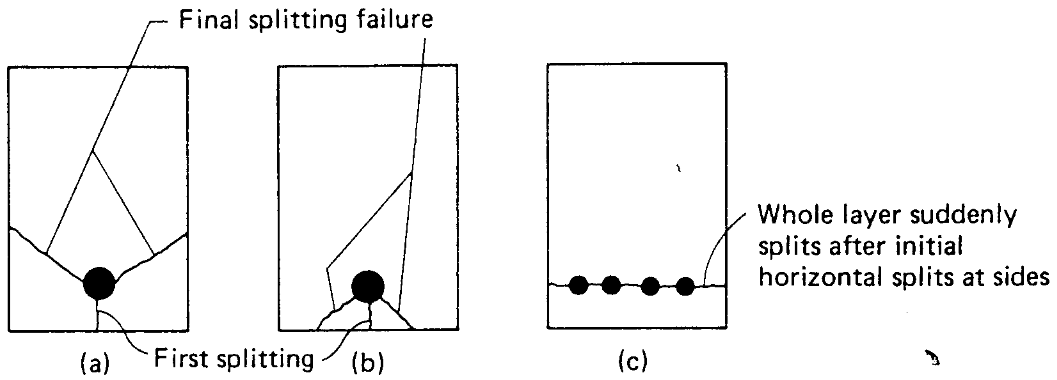


Figure 6.4.2 Splitting cracks and ultimate splitting failure modes (from ACI Committee 408 [6.1]).

Untrauer and Warren [6.16], Kemp and Wilhelm [6.17], Morita and Kaku [6.18], Jimenez, White, and Gergely [6.19], Kemp [6.7], Mirza [6.20], Moehle, Wallace, and Hwang [6.21], Darwin, McCabe, Idun, Schoenekase [6.22], Lutz, Mirza, and Gosain [6.23], and Hwang, Leu, and Hwang [6.24].

The studies of Orangun, Jirsa, and Breen [6.3] and Untrauer and Warren [6.16] have hypothesized that the action of splitting arises from a stress condition analogous to a concrete cylinder surrounding a reinforcing bar and acted upon by the outward radial components [Fig. 6.4.1(c)] of the bearing forces from the bar. The cylinder would have an inner diameter equal to the bar diameter d_b and a thickness C equal to the smaller of C_b, the clear bottom cover, or C_s, half of the clear spacing to the next adjacent bar (see Fig. 6.4.3). The tensile strength of this concrete cylinder determines the resistance against splitting. If $C_s < C_b$, a side-split type of failure occurs [Fig. 6.4.2(c)]. When $C_s > C_b$, longitudinal cracks through the bottom cover form first [first splitting cracks in Fig. 6.4.2(a), (b)]. If C_s is only nominally greater than C_b, the secondary splitting will be side splitting along the plane of the bars. If C_s is significantly greater than C_b, the secondary splitting will also be through the bottom cover to create a V-notch failure [Fig. 6.4.2(b)].

The proposal of ACI Committee 408 [6.5, 6.25] recognizes the cylinder hypothesis for splitting failure. The portion of the proposal relating to hooks (see Section 6.11) was adopted for the 1983 ACI Code, and the portion relating to straight bar development length formed the basis for the relatively complex

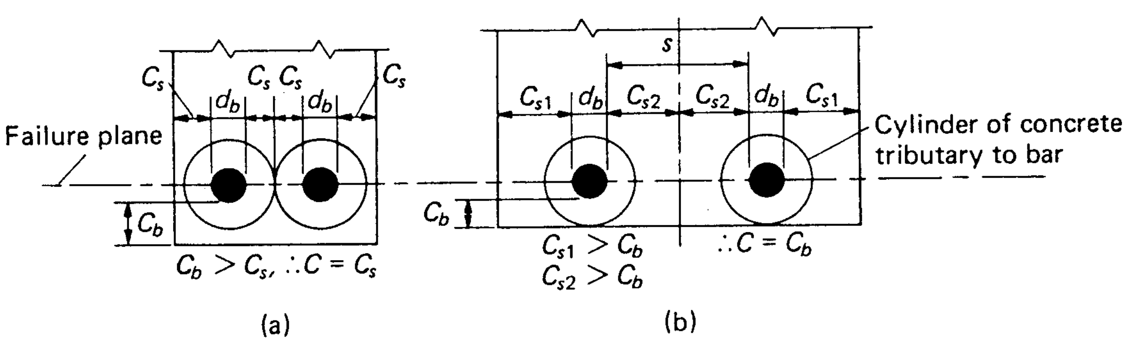


Figure 6.4.3 Concrete cylinder hypothesis for splitting failure (from Orangun, Jirsa, and Breen [6.3]).

Thus, the localized situation, relating to rate of change in moment, does not directly correlate with the development-length-related strength of the member. When the bars are properly anchored, that is, they have adequate development length provided and continue to carry their required tensile force, the localized stress condition is not of concern.

6.6 Moment Capacity Diagram—Bar Bends and Cutoffs

As stated in Section 6.2, the moment capacity of a beam at any section along its length is a function of its cross-section and the actual embedment length of its reinforcement. The concept of a diagram showing this three-dimensional relationship can be a valuable aid in determining cutoff or bend points of longitudinal reinforcement. It may be recalled from Chapter 3 that in terms of the cross section, the moment capacity (i.e., strength) for a singly reinforced rectangular may be expressed

$$M_n = A_s f_y (d - a/2) \qquad \textbf{[3.8.1]}$$

Equation (3.8.1) assumes that the steel reinforcement comprising A_s is adequately embedded *in each direction* by the required development length L_d from the section where M_n is computed such that the stress f_y is reached.

■ **EXAMPLE 6.6.1** Compute and draw the moment capacity diagram qualitatively for the beam of Fig. 6.6.1.

Solution: The procedure is basically the same whether strength (M_n or ϕM_n) or working stress moment capacity is desired.

The maximum capacity in each region is represented by the horizontal portions of the diagram in Fig. 6.6.1. In this example, there are five bars of one size in section *C–C*; thus the maximum moment capacity represented by each bar is in this case approximately one-fifth of the total capacity. Actually, the sections with four and two bars will have a little more than four-fifths and two-fifths, respectively, of the total capacity of the section containing five bars, due to the slight increase in moment arm when the number of bars in the section decreases.

At point a, the location where the fifth bar terminates, this bar has zero embedment length to the left and thus has zero capacity. Proceeding to the right from point a, the bar may be counted on to carry a tensile force proportional to its embedment from point a up to the development length L_d. Thus, in Fig. 6.6.1, point b represents the point where the fifth bar is fully developed through the distance L_d and can therefore carry its full tensile capacity. The other cutoff points are treated in the same way. ■

■ **EXAMPLE 6.6.2** Demonstrate qualitatively the practical use of the moment capacity ϕM_n diagram for verification of the locations of cutoff or bend points in a design. Assume that the main cross-section with five equal-sized bars provides exactly the required strength at midspan for this simply supported beam with uniform load, as shown in Fig. 6.6.2.

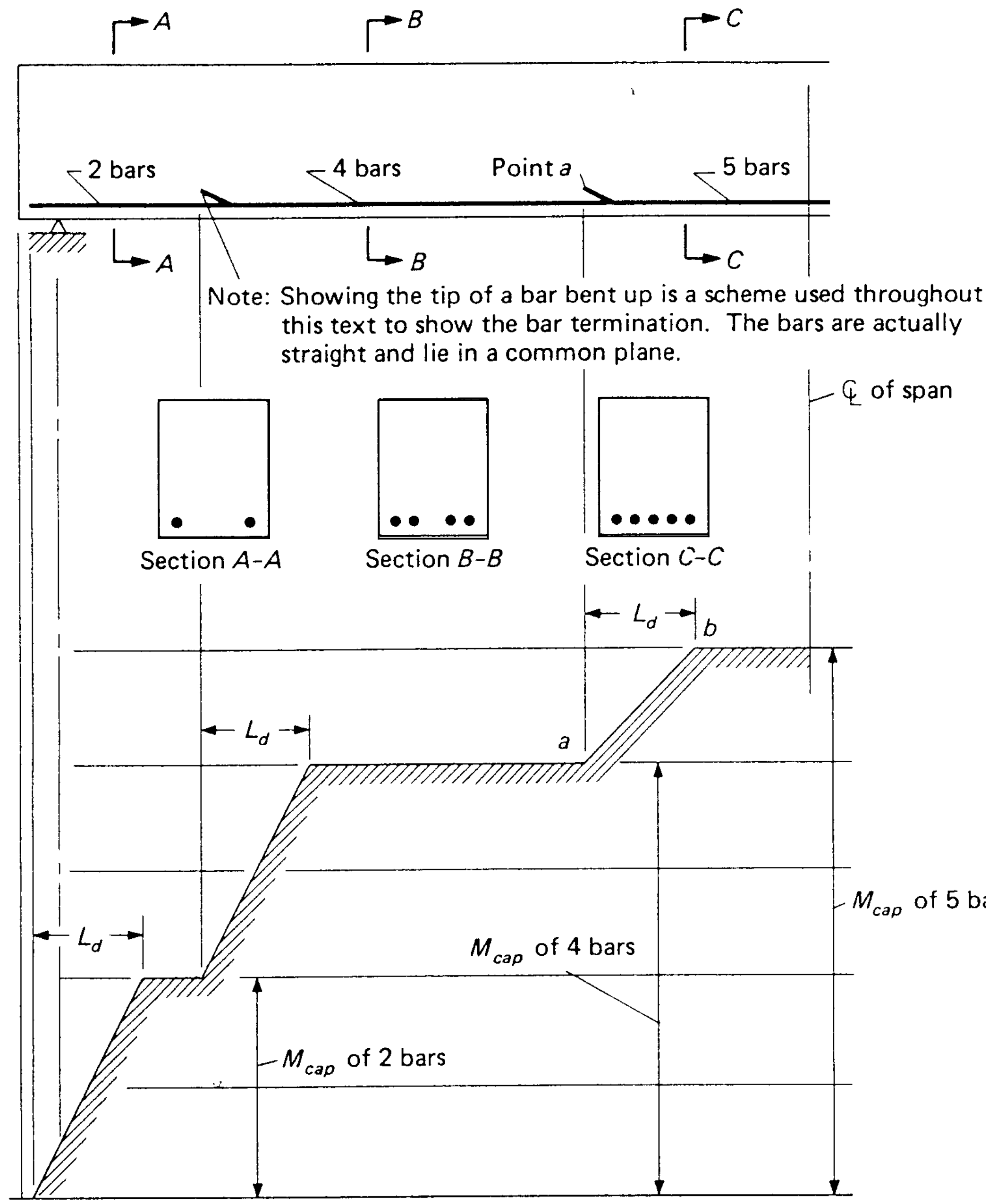
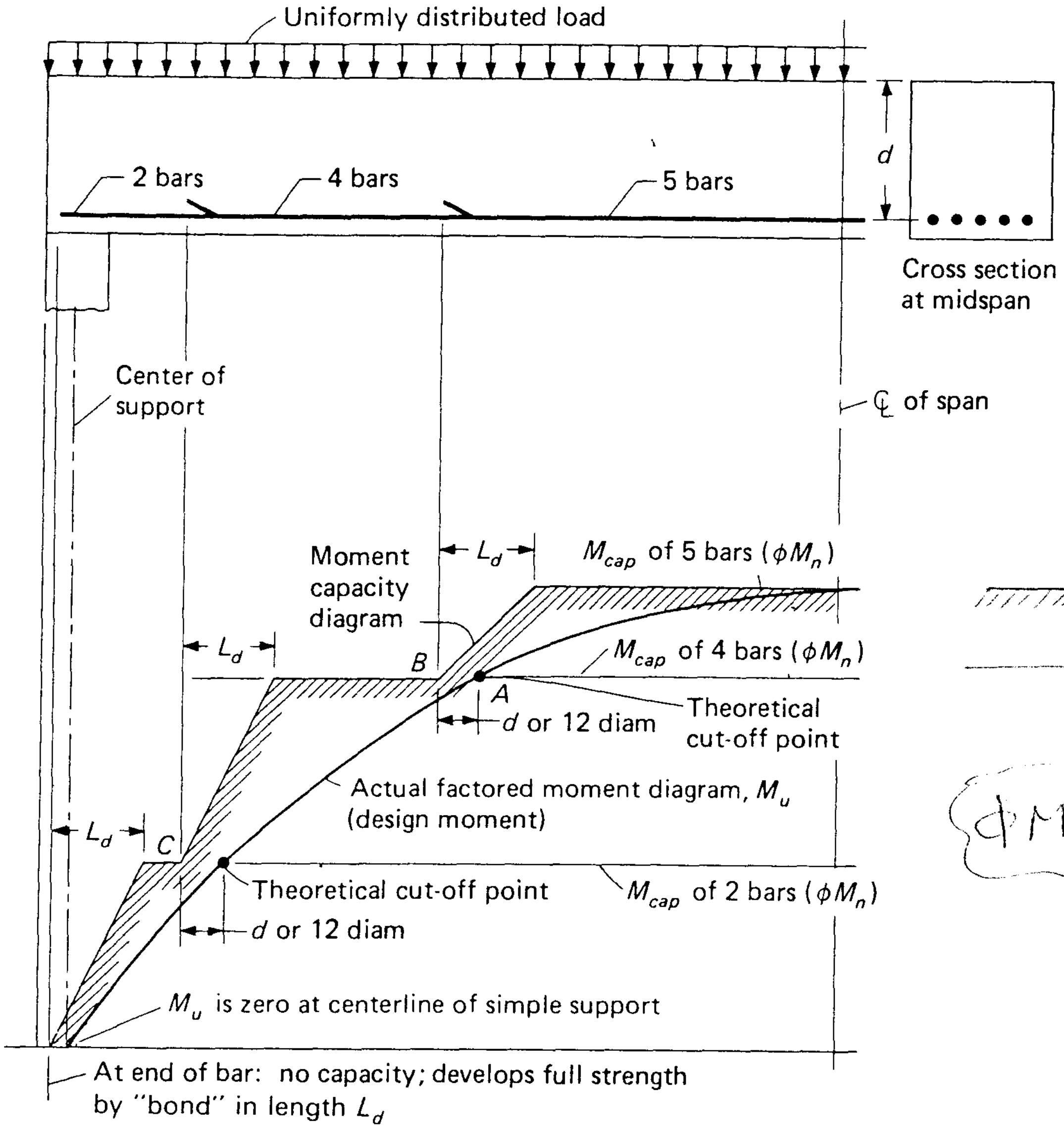


Figure 6.6.1 Moment capacity diagram.

Solution: **(a)** Compute the actual ϕM_n for each potential bar grouping that may be used; in the present case, for five bars, four bars, and two bars.

(b) Decide which bars must extend entirely across the span and into the support. ACI-12.11.1 states that "At least one-third the positive moment reinforcement in simple members . . . shall extend along the same face of member into the support." In beams, the reinforcement must extend into the support at least 6 in. In this case, two bars should extend into the support.

(c) Decide on the order of cutting or bending the remaining bars. The least amount of longitudinal reinforcement will be obtained when the resulting moment capacity ϕM_n diagram is closest to the factored moment M_u diagram. With that thought in mind, and proceeding from maximum moment region to the support, cut off one bar as soon as permissible.



ϕM_n

M_u

$\phi M_n \geq M_u$

Figure 6.6.2 Verification of bar cutoffs with the moment capacity diagram.

(d) Cutoff restrictions. Point A of Fig. 6.6.2 is the theoretical location to the left of which the capacity represented by the remaining four bars is adequate. To provide for a safety factor against shifting of the moment M_u diagram (especially in continuous spans) and to provide partially for the complexity arising from a potential diagonal crack, the ACI Code provides that there must be an extension beyond the point where a bar theoretically may be terminated, or it may be bent into the compression face. In ACI-12.10.3 is the statement, "Reinforcement shall extend beyond the point at which it is no longer required to resist flexure for a distance equal to the effective depth of the member or 12 bar diameters, whichever is greater, except at supports of simple spans and at free end of cantilevers."

(e) Once cutoff or bend points are located, a check is made by drawing the moment capacity ϕM_n diagram to ensure no encroachment on the factored moment M_u diagram.

(f) Other restrictions. Since points B and C of Fig. 6.6.2 are bar terminations in a tension zone, the stress concentrations described in Section 6.5 are present, effectively reducing the shear strength of the beam [6.26, 6.27]. Thus, *one* of the three special conditions of ACI-12.10.5 must be satisfied for cutoffs to be acceptable. However, if these bars were bent up and anchored in the compression zone, no further investigation would be necessary. ■

6.7 Development Length for Tension Reinforcement —ACI Code

The term "development length" has been defined in Sec. 6.2 as the length of embedment needed to develop the yield stress in the reinforcement. As described in Section 6.4, the development length requirement is primarily a function of the splitting resistance of the concrete surrounding the bars rather than a frictional-adhesional pullout resistance. The splitting resistance is roughly proportional to the bar area, indicated by Eq. (6.2.2); whereas the pullout resistance is roughly proportional to the bar diameter, indicated by Eq. (6.2.1).

In the 1989 ACI Code, completely new bar development provisions were adopted (ACI-12.2), recognizing the effects of (a) lateral spacing of bars being developed, (b) clear cover over bars being developed, and (c) confinement, if any, by stirrups, ties, or spirals around the bars being developed. Those provisions are described in detail in the 5th edition of this text, and are not repeated here.

Because of the seeming complexity of the 1989 provisions for bar development, and in response to strong encouragement from the profession, ACI Committee 318 revised the requirements for the 1995 ACI Code.

The 1995 Code provisions are based on the same basic relationship developed by Orangun, Jirsa, and Breen [6.3, 6.4] that formed the basis for the 1989 Code provisions. The 1995 provisions are also influenced by a more recent study Sozen and Moehle [6.28].

The general equation, after some tampering with the Orangun, Jirsa, and Breen [6.3, 6.4] format, is given in ACI-12.2.3 as ACI Formula (12-1),

$$\frac{L_d}{d_b} = \frac{3}{40}\frac{f_y}{\sqrt{f'_c}}\frac{\alpha\beta\gamma\lambda}{\left(\dfrac{c + K_{tr}}{d_b}\right)} \tag{6.7.1*}$$

where

L_d = development length

d_b = nominal diameter of bar or wire

c = cover or spacing dimension

= the smaller of (1) distance from center of bar being developed to the nearest concrete surface, and (2) one-half the center-to-center spacing of bars being developed

---

* For SI, with f_y and f'_c in MPa,

$$\frac{L_d}{d_b} = \frac{15}{16}\frac{f_y}{\sqrt{f'_c}}\frac{\alpha\beta\gamma\lambda}{\left(\dfrac{c + K_{tr}}{d_b}\right)} \tag{6.7.1}$$

The transverse reinforcement term K_{tr} is defined as follows:

$$K_{tr} = \frac{A_{tr} f_{yt}}{1500sn} \tag{6.7.2}^*$$

where

A_{tr} = total cross-sectional area of all transverse reinforcement which is within the spacing s and which crosses the potential plane of splitting through the reinforcement being developed

f_{yt} = specified yield strength of transverse reinforcement, psi

s = maximum center-to-center spacing of transverse reinforcement within development length L_d

n = number of bars being developed along the plane of splitting

In the use of Eq. (6.7.1), the cover and transverse reinforcement term cannot be taken greater than 2.5; thus,

$$\left(\frac{c + K_{tr}}{d_b}\right) \leq 2.5 \tag{6.7.3}$$

The symbols α, β, γ and λ in Eq. (6.7.1) represent the following modification factors:

α = modification factor for reinforcement location
= 1.3 for top bars[†]
= 1.0 for other bars

β = modification factor for epoxy-coated reinforcement
= 1.5 when cover $< 3d_b$ or clear spacing $< 6d_b$
= 1.2 other epoxy-coated reinforcement
= 1.0 non-epoxy-coated reinforcement

$\alpha\beta$ = need not exceed 1.7

γ = modification factor for bar size
= 0.8 for #6 and smaller bars and deformed wire
= 1.0 for #7 and larger bars

λ = modification factor for lightweight aggregate concrete
= 1.3 for lightweight aggregate concrete
(or $6.7\sqrt{f'_c}/f_{ct} \geq 1.0$ when f_{ct} is specified)
= 1.0 for normal-weight concrete

---

*For SI, with f_{yt} in MPa,

$$K_{tr} = \frac{A_{tr} f_{yt}}{260sn} \tag{6.7.2}$$

[†]Top bars are defined in ACI-12.2.4 as "Horizontal reinforcement so placed that more than 12 in. of fresh concrete is cast in the member below the development length or splice."

Simplified Equations of ACI-12.2.2. The use of Eq. (6.7.1) is clearly a complicated way of determining development length L_d. For most practical situations the simplified expression of ACI-12.2.2 can be used. The simplified equations divide into two categories, as follows:

Category A. Either one of the following two conditions will satisfy this most favorable situation:

1. **(a)** Clear lateral spacing between bars at least d_b, *and* ≥ d_b D (p. 67)
   (b) Clear cover at least d_b, *and*
   (c) minimum stirrups or ties along the development length (as per ACI-7.10.5 for ties, or ACI-11.5.4 combined with ACI-11.5.5.3 for stirrups)

or 2. **(a)** clear lateral spacing between bars at least $2d_b$, *and*
   (b) clear cover not less than d_b

The L_d/d_b simplification of Eq. (6.7.1) after substituting $(c + K_{tr})/d_b = 1.5$ becomes:

For #6 and smaller bars:

$$\frac{L_d}{d_b} = \frac{f_y}{25\sqrt{f'_c}}\alpha\beta\lambda \qquad \textbf{(6.7.4)*}$$

For #7 and larger bars:

$$\frac{L_d}{d_b} = \frac{f_y}{20\sqrt{f'_c}}\alpha\beta\lambda \qquad \textbf{(6.7.5)†}$$

Category B. Anything not in Category A is in Category B. The L_d/d_b simplification of Eq. (6.7.1) after substituting $(c + K_{tr})/d_b = 1.0$ becomes

For #6 and smaller bars:

$$\frac{L_d}{d_b} = \frac{3f_y}{50\sqrt{f'_c}}\alpha\beta\lambda \qquad \textbf{(6.7.6)‡}$$

For #7 and larger bars:

----

*For SI, with f_y and f'_c in MPa, for #20M (see Table 1.12.2) and smaller:

$$\frac{L_d}{d_b} = \frac{f_y}{2\sqrt{f'_c}}\alpha\beta\lambda \qquad \textbf{(6.7.4)}$$

†For SI, with f_y and f'_c in MPa, for #25M (see Table 1.12.2) and larger:

$$\frac{L_d}{d_b} = \frac{5f_y}{8\sqrt{f'_c}}\alpha\beta\lambda \qquad \textbf{(6.7.5)}$$

‡For SI, with f_y and f'_c in MPa, for #20M and smaller:

$$\frac{L_d}{d_b} = \frac{3f_y}{4\sqrt{f'_c}}\alpha\beta\lambda \qquad \textbf{(6.7.6)}$$

$$\frac{L_d}{d_b} = \frac{3f_y}{40\sqrt{f'_c}}\alpha\beta\lambda \qquad (6.7.7)^\dagger$$

Useful Tables. Development length for common values of f_y and f'_c are shown in Tables 6.7.1 to 6.7.4 for all bar sizes. Note the descriptive heading on the top of these tables, especially the reference to $\alpha\beta\lambda$.

Table 6.7.1 Development Length for Category A*, Eqs. (6.7.4) and (6.7.5) with $\alpha\beta\lambda = 1.0$

	INCH-POUND BARS WITH L_d IN INCHES					
	f_y = 40,000 psi			f_y = 60,000 psi		
	f'_c (psi)			f'_c (psi)		
BAR	3000	4000	5000	3000	4000	5000
(6.7.4) * #3	12.0	12.0	12.0	16.4	14.2	12.7
#4	14.6	12.6	12.0	21.9	19.0	17.0
#5	18.3	15.8	14.1	27.4	23.7	21.2
#6	21.9	19.0	17.0	32.9	28.5	25.5
#7	32.0	27.7	24.7	47.9	41.5	37.1
#8	36.5	31.6	28.3	54.8	47.4	42.4
#9	41.2	35.7	31.9	61.8	53.5	47.9
(6.7.5) † #10	46.4	40.2	35.9	69.6	60.2	53.9
#11	51.5	44.6	39.9	77.2	66.9	59.8
#14	61.8	53.5	47.9	92.7	80.3	71.8
#18	82.4	71.4	63.8	124	107	95.8
	CANADIAN METRIC BARS WITH L_d IN CENTIMETERS					
	f_y = 300 MPa			f_y = 400 MPa		
	f'_c (MPa)			f'_c (MPa)		
BAR	25	30	35	25	30	35
(6.7.4) #10M	33.9	30.9	30.0	45.2	41.3	38.2
#15M	48.0	43.8	40.6	64.0	58.4	54.1
#20M	58.5	53.4	49.4	78.0	71.2	65.9
#25M	94.5	86.3	79.9	126	115	106
#30M	112	102	94.8	150	136	126
#35M	134	122	113	179	163	151
(6.7.5) #45M	164	150	138	219	199	185
#55M	212	193	179	282	257	238

*(a) Clear spacing and clear cover $\geq d_b$ and minimum stirrups, ***or***
(b) clear spacing $\geq 2d_b$ and clear cover $\geq d_b$

Continued Use of 1989 ACI Code Procedure. Because the 1995 Code procedure is based on the same research background as for the 1989 Code procedure, the ACI Commentary (ACI-R12.2) states "Thus, design aids and computer programs

---

†For SI, with f_y and f'_c in MPa, for #25M and larger:

$$\frac{L_d}{d_b} = \frac{15f_y}{16\sqrt{f'_c}}\alpha\beta\lambda \qquad (6.7.7)$$

Table 6.7.2 Development Length for Category B*, Eqs. (6.7.6) and (6.7.7) with $\alpha\beta\lambda = 1.0$

	INCH-POUND BARS WITH L_d IN INCHES					
	f_y = 40,000 psi			f_y = 60,000 psi		
	f'_c (psi)			f'_c (psi)		
BAR	3000	4000	5000	3000	4000	5000
#3	16.4	14.2	12.7	24.6	21.3	19.1
#4	21.9	19.0	17.0	32.9	28.5	25.5
#5	27.4	23.7	21.2	41.1	35.6	31.8
#6	32.9	28.5	25.5	49.3	42.7	38.2
#7	47.9	41.5	37.1	71.9	62.3	55.7
#8	54.8	47.4	42.4	82.2	71.2	63.6
#9	61.8	53.5	47.9	92.7	80.3	71.8
#10	69.6	60.2	53.9	104	90.4	80.8
#11	77.2	66.9	59.8	116	100	89.7
#14	92.7	80.3	71.8	139	120	108
#18	124	107	95.8	185	161	144
	CANADIAN METRIC BARS WITH L_d IN CENTIMETERS					
	f_y = 300 MPa			f_y = 400 MPa		
	f'_c (MPa)			f'_c (MPa)		
BAR	25	30	35	25	30	35
#10M	50.9	46.4	43.0	67.8	61.9	57.3
#15M	72.0	65.7	60.9	96.0	87.6	81.1
#20M	87.8	80.1	74.2	117	107	98.9
#25M	142	129	120	189	173	160
#30M	135	154	142	224	205	190
#35M	161	183	170	268	244	226
#45M	197	224	208	328	299	277
#55M	254	290	268	423	386	358

*Everything not in Category A.

based on Section 12.2 of ACI 318-89 can be used for complying with the 1995 ACI Building Code." The authors of this text have not included those 1989 provisions (they are in the 5th edition) believing that the above simplified equations, along with an optional more "exact" equation, will be strongly preferred by most users.

In brief, the 1989 Code (ACI-12.2) required computation of a basic development length based on splitting strength. The basic development was modified by multiplying by 1.0, 1.4, or 2.0; the larger value for the most unfavorable bar spacing and cover condition. That modified basic length could not be less than the minimum development length based on pullout, $0.03 d_b f_y / \sqrt{f'_c}$. That gave the "first-level L_d"; it was then multiplied by "second-level" modifications (1) α for top bars, (2) β for epoxy-coated bars, (3) λ for lightweight aggregate concrete, and (4) α_{exs} for excess reinforcement. Final result could not be less than 12 in.

The maximum value on the cover and transverse reinforcement term, $(c + K_{tr})/d_b$, of 2.5 in the 1995 ACI Code is intended to maintain the same minimum development length as prescribed in the 1989 ACI Code based on a pullout failure mode.

Table 6.7.3 Development Length for Category A*, Eqs. (6.7.4) and (6.7.5) with $\alpha\beta\lambda = 1.0$

1996 ASTM METRIC BARS WITH L_d IN CENTIMETERS

	$f_y = 300$ MPa			$f_y = 420$ MPa		
	f'_c (MPa)			f'_c (MPa)		
BAR	25	30	35	25	30	35
#10M	30.0	30.0	30.0	39.9	36.4	33.7
#13M	38.1	34.8	32.2	53.3	48.7	45.1
#16M	47.7	43.5	40.3	66.8	61.0	56.4
#19M	57.3	52.3	48.4	80.2	73.2	67.8
#22M	83.3	76.0	70.4	117	106	98.5
#25M	95.3	87.0	80.5	133	122	113
#29M	108	98.2	91.0	151	138	127
#32M	121	111	102	170	155	143
#36M	134	123	113	188	172	159
#43M	161	147	136	226	206	191
#57M	215	196	182	301	275	254

*(a) Clear spacing and clear cover $\geq d_b$ and minimum stirrups, ***or***
(b) clear spacing $\geq 2d_b$ and clear cover $\geq d_b$

Table 6.7.4 Development Length for Category B*, Eqs. (6.7.6) and (6.7.7) with $\alpha\beta\lambda = 1.0$

1996 ASTM METRIC BARS WITH L_d IN CENTIMETERS

	$f_y = 300$ MPa			$f_y = 420$ MPa		
	f'_c (MPa)			f'_c (MPa)		
BAR	25	30	35	25	30	35
#10M	42.8	39.0	36.1	59.9	54.6	50.6
#13M	57.2	52.2	48.3	80.0	73.0	67.6
#16M	71.6	65.3	60.5	100	91.4	84.7
#19M	86.0	78.5	72.6	120	110	102
#22M	125	114	106	175	160	148
#25M	143	130	121	200	183	169
#29M	161	147	136	226	206	191
#32M	182	166	154	254	232	215
#36M	201	184	170	282	257	238
#43M	242	221	204	339	309	286
#57M	322	294	272	451	412	381

*Everything not in Category A.

Practical Application of ACI-12.2 Development Length Rules. The practicality for applying the rules in ordinary reinforced concrete construction is that most beams will contain at least ACI Code-specified minimum stirrups (thereby satisfying Category A, item 1c), clear spacing must satisfy the larger of the bar diameter d_b or 1 in. (ACI-7.6.1), and cover must satisfy the minimum specified in ACI-7.7.1 in any case. Using the minimum 1.5 in. of cover on beams will commonly provide the Category A minimum of d_b. For slab-like elements without shear reinforcement, clear spacing will usually satisfy the Category A, item 2a,

■ **EXAMPLE 6.8.1** Determine the development length L_d required for the #9 epoxy-coated bars A on the top of a 15-in. slab, as shown in Fig. 6.8.1. Use f_y = 60,000 psi, and f'_c = 4000 psi with lightweight aggregate concrete.

Solution: **(a)** Determine the development length L_d using the simplified equations. Since cover of 1.5 in. exceeds d_b of 1.128 in., and the 8 in. bar spacing exceeds clear spacing of $2d_b$ (i.e., 2.3 in.), the situation is Category A, item 2, and for #9 bars Eq. (6.7.5) applies,

$$\frac{L_d}{d_b} = \frac{f_y}{20\sqrt{f'_c}}\alpha\beta\lambda \qquad [6.7.5]$$

$$= \frac{60{,}000}{20\sqrt{4000}}\alpha\beta\lambda = 47.4\alpha\beta\lambda$$

$$L_d = 47.4d_b\alpha\beta\lambda = 47.4(1.128)\alpha\beta\lambda = 53.5\alpha\beta\lambda$$

Note that 53.5 in. agrees with the value in Table 6.7.1.

Referring to Fig. 6.8.1, when checking the bar spacing, bars A are developed over distance 1-2, while bars B are developed over the distance 2-3. The spacing

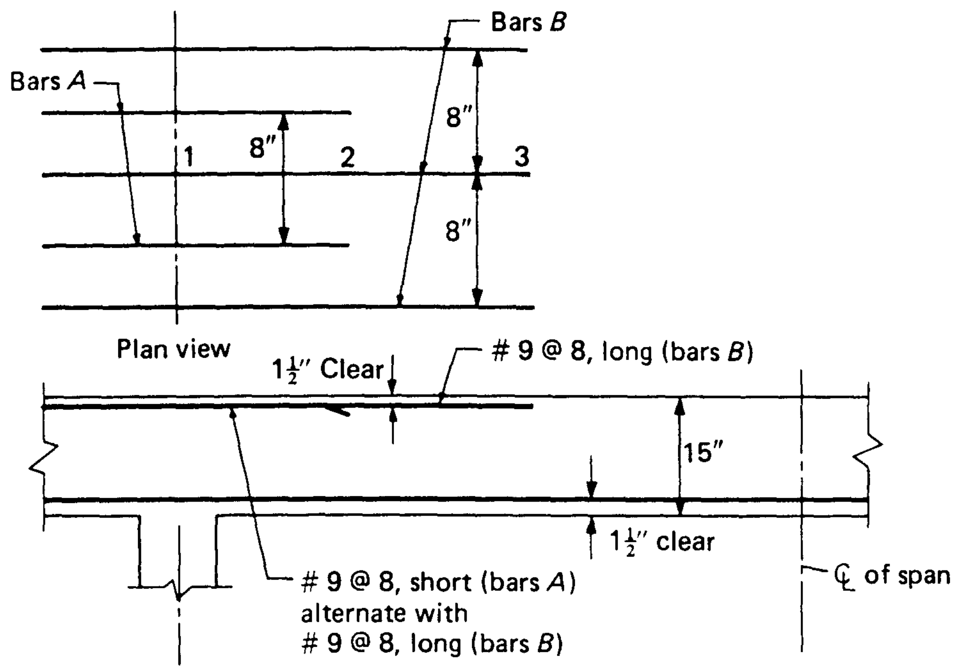


Figure 6.8.1 Top bars for Example 6.8.1.

to be used for bars A is the spacing of the closest bars that terminate at the same point. In other words, the spacing for both bars A and B is 8 in.

(b) Modification α for top bars. Since the negative moment region bars are cast with more than 12 in. of fresh concrete below them, they are top bars according to ACI-12.2.4; thus $\alpha = 1.3$.

(c) Modification β for epoxy-coated bars. Check clear cover,

$$\text{Clear cover} = \frac{1.5}{d_b} = \frac{1.5}{1.128} = 1.3d_b < 3d_b$$

Since clear cover is less than $3d_b$, $\beta = 1.5$. The maximum value of $\alpha\beta = 1.7$.

(d) Modification λ for lightweight aggregate concrete. The lightweight aggregate concrete multiplier $\lambda = 1.3$.

(e) Final development length L_d.

$$L_d = 53.5\alpha\beta\lambda = 53.5(1.7)1.3 = 118 \text{ in}\ .$$

(f) Compute development length L_d. Using the general equation, Eq. (6.7.1).

$$\frac{L_d}{d_b} = \frac{3}{40}\frac{f_y}{\sqrt{f'_c}}\frac{\alpha\beta\gamma\lambda}{\left(\dfrac{c + K_{tr}}{d_b}\right)} \qquad \textbf{[6.7.1]}$$

There are no stirrups; thus $K_{tr} = 0$. The value of c for Eq. (6.7.1) is the smaller of the cover (i.e., the distance from the center of the bar to the nearest concrete face) or one-half the center-to-center spacing of the bars being developed. In this case,

$$\text{Cover} = 1.5 + 1.128/2 = 2.06 \text{ in.}$$

$$(\text{Center-to-center spacing})/2 = 8/2 = 4 \text{ in.}$$

Thus,
$$\left(\frac{c + K_{tr}}{d_b} = \frac{2.06 + 0}{1.128} = 1.83\right) \leq 2.5$$

[6.7.3]

$$\frac{L_d}{d_b} = \frac{3}{40}\frac{f_y}{\sqrt{f'_c}}\frac{\alpha\beta\gamma\lambda}{\left(\dfrac{c + K_{tr}}{d_b}\right)}$$

$$= \frac{3}{40}\frac{60{,}000}{\sqrt{4000}}\frac{(1.7)(1.0)1.3}{1.83} = 71.1\frac{(1.7)(1.0)1.3}{1.83} = 85.9$$

In the above calculation, $\alpha\beta = 1.7$, the upper limit of that product, which exceeds the actual $\alpha\beta = 1.3(1.5) = 1.95$. The bar size factor $\lambda = 1.0$ for #7 bars and larger. Thus,

$$L_d = 85.9d_b = 85.9(1.128) = 96.9 \text{ in.}$$

The simplified method gave $L_d = 118$ in. That formula used $(c + K_{tr})/d_b = 1.5$, whereas Eq. (6.7.3) gave 1.83, thus giving the more accurate L_d as 82% of the value from the simplified equation. ■

6.9 Development Length for Compression Reinforcement

Relatively less is known about the development length for compression bars than for tension bars, except that the weakening effect of flexural tension cracks is not present and there is beneficial effect of the end bearing of the bars on the concrete. ACI-12.3 gives as the basic development length L_{db},

$$L_{db} = 0.02d_b\frac{f_y}{\sqrt{f'_c}} \qquad \textbf{(6.9.1)*}$$

which is basically two-thirds of the minimum development length for tension reinforcement to prevent a "pullout" mode of failure. ACI-12.3 also states that L_{db} must not be less than

$$L_{db} \geq 0.0003d_bf_y \qquad \textbf{(6.9.2)*}$$

which means that only f'_c up to about 4400 psi may be counted upon. Thus the basic development length L_{db} is to be taken as the larger of Eqs. (6.9.1) and (6.9.2).

When excess bar area is provided such that provided A_s exceeds required A_s, Eqs. (6.9.1) or (6.9.2), whichever controls, may be reduced by applying the multiplier (required A_s/provided A_s).

Reduction in development length is permitted when reinforcement is enclosed by spirals or closely spaced ties (typically in columns; see Chapter 13) which are not less than $\frac{1}{4}$-in. diameter for spirals (ACI-7.10.4), or #4 bars for ties (ACI-7.10.5), and having a pitch (for spirals) or center-to-center spacing (for ties) not exceeding 4 in. Under these confinement conditions, L_{db} may be reduced 25%.

---

*For SI, ACI 318–95M, for L_{db} and d_b in mm, and f'_c and f_y in MPa, gives

$$L_{db} = 24d_b\frac{f_y}{\sqrt{f'_c}} \qquad \textbf{(6.9.1)}$$

$$L_{db} = 0.044\ d_bf_y \qquad \textbf{(6.9.2)}$$

After all modifications, the development length L_d is not permitted to be less than 8 in. (200 mm). Thus, in general, for compression reinforcement.

$$L_d = \begin{bmatrix} \text{Eqs. (6.9.1)} \\ \text{or (6.9.2)} \end{bmatrix} \left[\frac{\text{required } A_s}{\text{provided } A_s} \right] \begin{bmatrix} 0.75 \text{ for enclosure} \\ \text{by spirals or ties} \end{bmatrix} \geq 8 \text{ in.} \quad \textbf{(6.9.3)}$$

■ **EXAMPLE 6.14.1** For the cantilever beam shown in Fig. 6.14.2 determine the distance L_1 from the support to the point where 2-#8 bars may be cut off. Assume the #4 stirrups shown (solid, not the dashed ones) have been preliminarily designed. Assume there will be at least L_d embedment of the bars into the support. Draw the resulting moment capacity ϕM_n diagram for the entire beam. Use $f_c' =$ 3000 psi and f_y = 60,000 psi.

Solution: **(a)** Compute the maximum moment capacity ϕM_n of the section.

$$0.75\rho_b \text{(Table 3.6.1)} = 0.0160$$

$$\rho = \frac{3(1.27) + 2(0.79)}{16(28)} = 0.0120 < 0.75\rho_b \qquad \textbf{OK}$$

$$C = 0.85(3)16a = 40.8a$$

$$T = [3(1.27) + 2(0.79)]60 = (3.81 + 1.58)60 = 323 \text{ kips}$$

$$a = \frac{323}{40.8} = 7.92 \text{ in.}$$

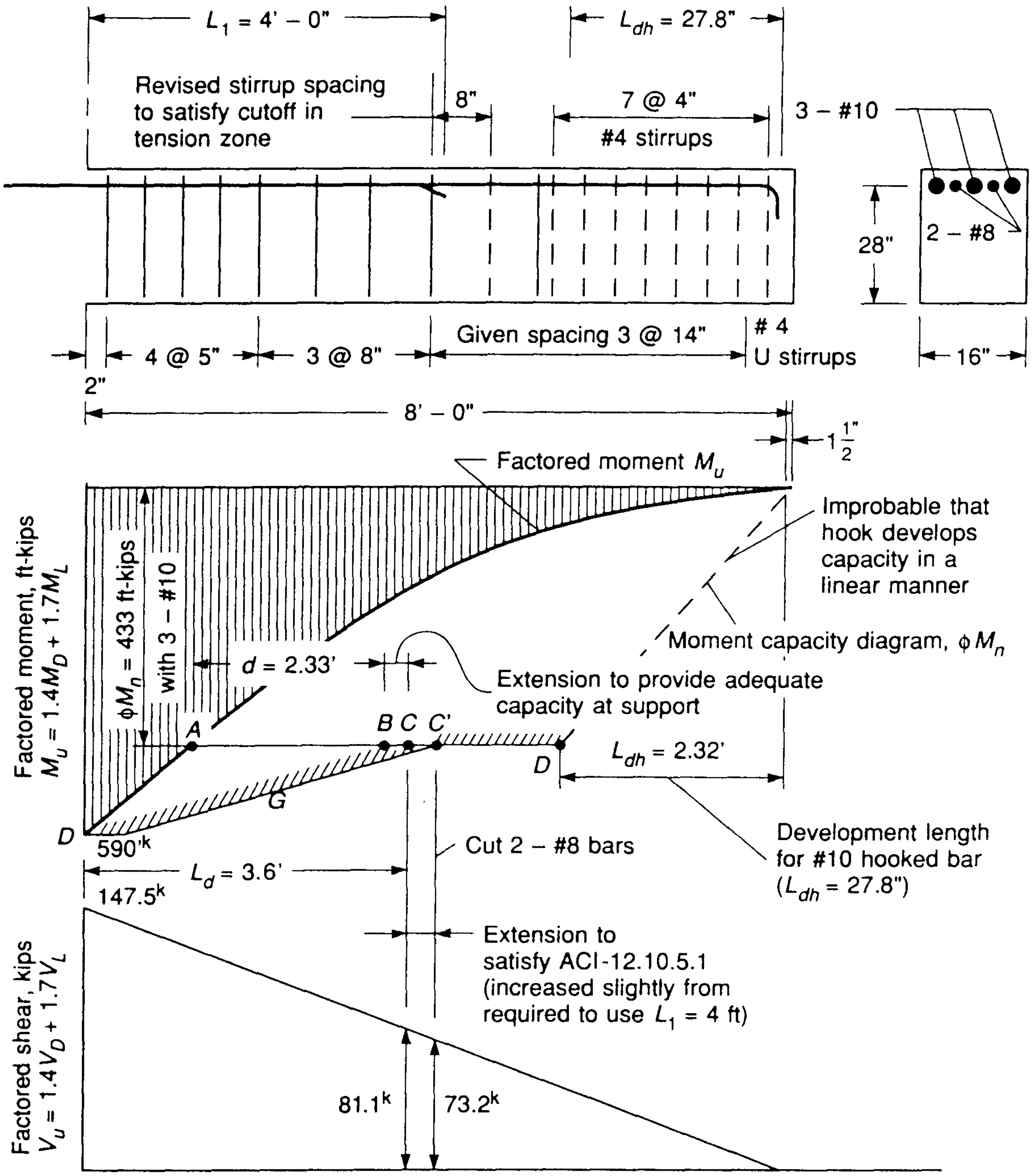


Figure 6.14.2 Beam of Example 6.14.1.

$$M_n = 323[28 - 0.5(7.92)]\tfrac{1}{12} = 647 \text{ ft-kips}$$

$$\phi M_n = 0.90(647) = 582 \text{ ft-kips} \approx M_u = 590 \text{ ft-kips} \qquad \textbf{OK}$$

(b) Determine the theoretical cutoff point for 2-#8 bars. The moment capacity ϕM_n remaining with 3-#10 bars is

$$C = 40.8a$$

$$T = 3.81(60) = 229 \text{ kips}$$

$$a = \frac{229}{40.8} = 5.61 \text{ in.}$$

$$\phi M_n = 0.90(229)[28 - 0.5(5.61)]\tfrac{1}{12} = 433 \text{ ft-kips}$$

Plot on the factored moment M_u diagram and locate the theoretical cutoff point A. Extend to the right 12 bar diameters (of the #8 bars that are to be cut) or the effective depth of the member, whichever is greater, to arrive at point B.

$$d = 28 \text{ in.} (2.33 \text{ ft}) > [12d_b = 12(1.0) = 12 \text{ in.}]$$

(c) Use the simplified equations to determine the development length L_d for #8 bars. Can Category A, the more favorable one, be used? Check the clear spacing of bars. Assuming the bars, though unequal in size, are uniformly spaced, the clear spacing between them is

$$\text{clear spacing} = \frac{16 - 2(1.5) - 2(0.5) - 3(1.27) - 2(1.0)}{4} = 1.55 \text{ in}$$

Since only the 2-#8 bars are being developed, and the 3-#10 are presumed to continue beyond the #8 cutoff location, it is the spacing between the two #8 that determines the Category. The failure mode would have splitting from a #8 bar to the side or top face of the member, or between the two #8 bars. The ACI Code rules consider a bar (or bars) as essentially inert when it is not being developed within the development region of other bars. Thus, when the #10 bars of this example have a development length from their termination near the free end of the cantilever that is less than the distance to the #8 bar cut, the #10 bars are considered to have no influence on L_d for the #8 bars. It is a matter of opinion whether or not the #10 itself should be treated as concrete. That is, in this case whether to use the full spacing between the #8 bars, 2(1.55) + 1.27 diam. of #10 = 4.37 in. The authors believe it appropriate in this case to consider the spacing of the #8 to be 4.37 in. for the purpose of satisfying a Category A requirement, *assuming L_d for the #10 does not overlap the L_d for the #8 bars.*

Even if the concrete width between #8 bars were taken as 2(1.55) = 3.10 in., it still exceeds the $2d_b$ for the #8 bar to satisfy Category A, item 2(a), given in Section 6.7 (ACI-12.2.2), as well as item 2(b), because cover to the top face of the beam is 2 in., which exceeds d_b needed for that item.

Thus, Category A applies! Using simplified Eq. (6.7.5) for #7 and larger bars

$$\frac{L_d}{d_b} = \frac{f_y}{20\sqrt{f'_c}}\alpha\beta\lambda \tag{6.7.5}$$

For the modification factors $\alpha\beta\lambda$, only the top bar factor $\alpha = 1.3$ applies. The epoxy-coated bar factor β and the lightweight aggregate concrete factor λ are both 1.0 because those factors do not apply. Thus, Eq. (6.7.5) gives

$$L_d = \frac{d_b f_y}{20\sqrt{f'_c}}\alpha\beta\lambda = \frac{1.0(60{,}000)}{20\sqrt{3000}}\alpha\beta\lambda$$

$$= 54.8\alpha\beta\lambda = 54.8(1.3)(1.0)1.0 = 71.2 \text{ in.}$$

The 54.8 in. can be verified from Table 6.7.1. Thus,

$$L_d(\text{for } \#8) = 71.2 \text{ in. } (5.9 \text{ ft})$$

This development length is 58% longer than the L_d of 3.75 ft obtained under the 1989 ACI Code, where the most favorable conditions applied.

(d) Use the general equation, Eq. (6.7.1), to determine the development length L_d for #8 bars. That equation is

$$\frac{L_d}{d_b} = \frac{3}{40}\frac{f_y}{\sqrt{f'_c}}\frac{\alpha\beta\gamma\lambda}{\left(\frac{c + K_{tr}}{d_b}\right)} \qquad \textbf{[6.7.1]}$$

The cover or spacing dimension c is the smaller of (1) distance from center of bar being developed to nearest concrete surface, and (2) one-half center-to-center spacing (*clear* spacing computed as 1.55 in. in part a) of bars being developed. The distance c is the smaller of the following two values:

$$\text{top and side cover} = 1.5(\text{i.e., clear}) + 0.5(\text{i.e., stirrup}) + 0.5(\text{i.e., bar radius}) = 2.5 \text{ in.}$$

$$\text{one-half center-to-center spacing} = 1.55 + 0.5(\text{i.e., bar radius}) = 2.05 \text{ in.}$$

Thus, $c = 2.05$ in.

For the stirrups in the development region, use the 8 in. spacing for computation. Use Eq. (6.7.2),

$$K_{tr} = \frac{A_{tr}f_{yt}}{1500sn} \qquad \textbf{[6.7.2]}$$

The number n of bars being developed is 2, and A_{tr} is the total area of stirrups surrounding the bars being developed, in this case, for #4 stirrups it is 2(0.2) times 8 stirrups. Thus, evaluation of Eq. (6.7.2) gives

$$K_{tr} = \frac{A_{tr}f_{yt}}{1500sn} = \frac{8(2)(0.20)60{,}000}{1500(8)2} = 8$$

Evaluating Eq. (6.7.3),

$$\left[\frac{c + K_{tr}}{d_b} = \frac{2.06 + 8.0}{1.0} = 10.1\right] > 2.5 \text{ max}$$

Thus, $(c + K_{tr})/d_b = 2.5$.

Evaluate Eq. (6.7.1),

$$L_d = \frac{3}{40}\frac{d_b f_y}{\sqrt{f'_c}}\frac{\alpha\beta\gamma\lambda}{\left(\frac{c + K_{tr}}{d_b}\right)}$$

$$= \frac{3}{40}\frac{(1.0)60{,}000}{\sqrt{3000}}\frac{\alpha\beta\gamma\lambda}{2.5} = 82.2\frac{1.3(1.0)(1.0)1.0}{2.5} = 42.7 \text{ in. } (3.6 \text{ ft})$$

The 42.7 in. compares favorably with the 1989 ACI Code value of 45.0 in., and is significantly lower than the 71.2 in. from the 1995 simplified equation. Use $L_d = 42.7$ in. for the moment capacity diagram in Fig. 6.14.2.

Since point B, the proposed cutoff point, lies only about 3.5 ft from the support, the #8 bars would not have full capacity at the support. Therefore, extend the proposed cutoff to point C, which is located at L_d (for #8) = 3.6 ft from the support.

(e) Check ACI-12.10.5 for cutting bars at point C in the tension zone. The shear strength, including contribution of stirrups, is first computed. Using the simplified method of constant V_c,

$$V_c = 2\sqrt{f'_c}\, b_w d = 2\sqrt{3000}\,(16)(28)\tfrac{1}{1000} = 49.1 \text{ kips}$$

For the 14-in. spaced #4 stirrups in the vicinity of the potential cut point C,

$$V_s = \frac{A_v f_y d}{s} = \frac{2(0.20)(60)28}{14} = 48.0 \text{ kips}$$

The shear strength ϕV_n at point C is

$$\phi V_n = \phi(V_c + V_s) = 0.85(49.1 + 48.0) = 82.5 \text{ kips}$$

$$\text{percent stressed in shear} = \frac{V_u}{\phi V_n} = \frac{81.1}{82.5} = 98\% > 75\% \qquad \textbf{NG}$$

Even when only 50% of the moment strength ϕM_n is used by M_u, the percent stressed in shear cannot exceed 75% (see Condition 3, Eqs. 6.12.3 and 6.12.4). Try using one more 8-in. stirrup spacing to cover the potential cut at point C, and see whether or not Condition 1, Eq. (6.12.1), is satisfied.

$$V_s = 48.0\left(\frac{14}{8}\right) = 84.0 \text{ kips}$$

$$\text{percent stressed in shear} = \frac{81.1}{0.85(49.1 + 84)} = 71\%$$

This is borderline to satisfy the two-thirds limit of ACI-12.10.5.1 (Eq. 6.12.1). Extend the #8 bars to point C' 4 ft from face of support.

(f) Check whether the continuing #10 bars have adequate development length to the right of point C. The clear spacing between the continuing three #10 bars is

$$\text{clear spacing} = \frac{16 - 2(1.5) - 2(0.5) - 3(1.27)}{2} = 4.1 \text{ in.}$$

which exceeds the $2d_b$ of 2.54 in. required for Category A, item 2(a). Top cover of 2.64 in. [i.e., $1.5 + 0.5 + 1.27/2) = 2.64$ in.] to the center of the #10 bars exceeds the d_b requirement of Category A, item 2(b). Thus, the simplified equation, Eq. (6.7.5) for #7 and larger bars,

$$L_d(\text{for \#10}) = \frac{d_b f_y}{20\sqrt{f'_c}}\alpha\beta\lambda = \frac{1.27(60{,}000)}{20\sqrt{3000}}\alpha\beta\lambda$$

$$= 69.6\alpha\beta\lambda = 69.6(1.3)(1.0)1.0 = 90.5 \text{ in. (7.5 ft)}$$

For the modification factors $\alpha\beta\lambda$, only the top bar factor $\alpha = 1.3$ applies.

Calculate the development length L_d based on the general equation, Eq. (6.7.1). The distance c is the smaller of the following two values:

top and side cover = 1.5(i.e., clear)
+ 0.5(i.e., stirrup) + 0.635(i.e., bar radius) = 2.6 in.

one-half center-to-center spacing = 4.1/2 + 0.635(i.e., bar radius) = 2.7 in.

Thus $c = 2.6$ in.

For the stirrups in the development region, use the given 14 in. spacing near the free end of the cantilever for computation. The number n of bars being developed is 3, and A_{tr} is the total area of stirrups surrounding the bars being developed, in this case, for #4 stirrups it is 2(0.2) times 3 stirrups. Use Eq. (6.7.2),

$$K_{tr} = \frac{A_{tr}f_{yt}}{1500sn} = \frac{3(2)(0.20)60{,}000}{1500(14)3} = 1.1$$

Evaluating Eq. (6.7.3),

$$\left[\frac{c + K_{tr}}{d_b} = \frac{2.6 + 1.0}{1.27} = 2.8\right] > 2.5 \text{ max}$$

Thus, $(c + K_{tr})/d_b = 2.5$.

Evaluate Eq. (6.7.1),

$$L_d = \frac{3}{40}\frac{d_b f_y}{\sqrt{f'_c}}\frac{\alpha\beta\gamma\lambda}{\left(\dfrac{c + K_{tr}}{d_b}\right)}$$

$$= \frac{3}{40}\frac{(1.27)60{,}000}{\sqrt{3000}}\frac{\alpha\beta\gamma\lambda}{2.5} = 104.3\frac{1.3(1.0)(1.0)1.0}{2.5} = 54.2 \text{ in. } (4.5 \text{ ft})$$

This embedment of 4.5 ft measured from the end of straight #10 bars would overlap the development length region of the #8 bars, possibly requiring longer development length L_d for the #8 bars because the center-to-center spacing then would be the reduced value based on five bars in the 16 in. width. However, in this case because K_{tr} is 8.0 [see part (d)] the value of $(c + K_{tr})/d_b$ remains at 2.5 and the L_d of #8 bars stands at 3.6 ft in part (d). The #10 bars would satisfy literally the statement of ACI-12.10.4, which requires "Continuing reinforcement shall have an embedment length not less than the development length L_d beyond the point where bent or terminated tension reinforcement is no longer required to resist flexure." In other words, the distance from point A to the free end of the cantilever must be at least L_d (for #10). The authors believe in a somewhat more conservative approach, requiring the moment capacity ϕM_n diagram to have an offset from the factored moment M_u diagram, except at or near a simple support or the free end of a cantilever, equal to 12 bar diameters of the effective length d, whichever is greater.

In this case, try standard 90° hooks (see Fig. 6.11.1) on the ends of the #10 bars. Since the beam has the usual 1.5-in. clear cover and #4 stirrups, the cover to the hooked bars is 2 in., which is less than the $2\frac{1}{2}$ in. required by ACI-12.5.4; thus, the special provisions of that Code section must be satisfied.

The development length L_{dh} for the #10 hooked bar is the basic value L_{hb} (i.e., no modification to L_{hb} applies) given by Eq. (6.11.1) and Table 6.11.2. Thus, for #10 hooked bar,

$$L_{dh} = L_{hb} = \frac{1200d_b}{\sqrt{f'_c}} = \frac{1200(1.27)}{\sqrt{3000}} = 27.8 \text{ in.}$$

which exceeds the minimum $8d_b$ or 6 in., whichever is greater (ACI-12.5.1). The L_{dh} of 27.8 in. is dimensioned from the outside face of the tail of the hook, as shown in Fig. 6.14.2. Of course, here stirrups spaced at not more than $3d_b$ (4.23 in.) must be provided along the 27.8 in. of development distance, in accordance with ACI-12.5.4.

(g) Moment capacity ϕM_n diagram. The full strength ϕM_n for the beam with 3-#10 hooked bars will be available at 27.8 in. from the outside of the hook on the end of the beam. Assuming 1.5 in. cover, full capacity is available at 29.3 in. (2.44 ft) from end of beam (point D). The dashed line in Fig. 6.14.2 has been drawn from zero strength at the end of the hook to full strength ϕM_n = 433 ft-kips at 2.44 ft from end of beam; however, it is *not* intended to imply that the hooked bar develops its strength linearly since that is highly improbable.

(h) Final decision. Cut 2-#8 bars at 4 ft-0 in. from the support; use 90° standard hooks on the 3-#10 bars; use 8-#4 U stirrups as confinement over the L_{dh} distance, as shown in Fig. 6.14.2 by the dashed stirrups.

The use of #10 bars in this cantilever beam is not a practical design but serves to illustrate the need for extending the cut location from B to C and then C', and the need for and treatment of hooked bars. ■