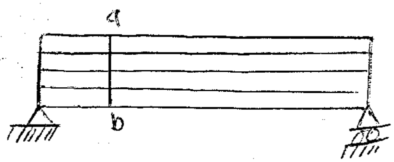
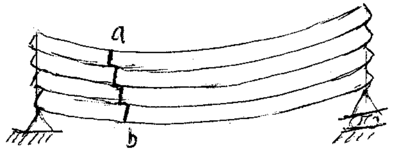
Chapter 6.

Shear Stress

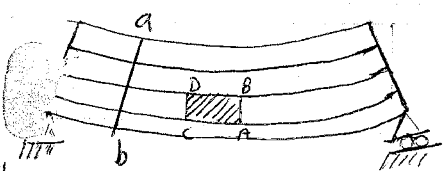
Shearing Stresses in Beams



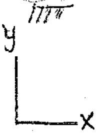
Laminated beam

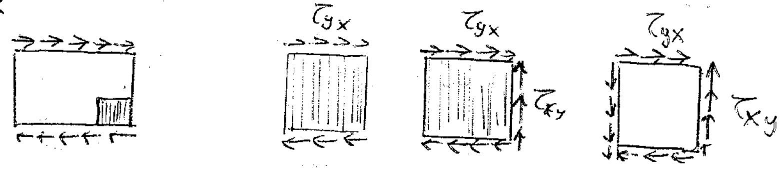


unglued lamina



glued lamina





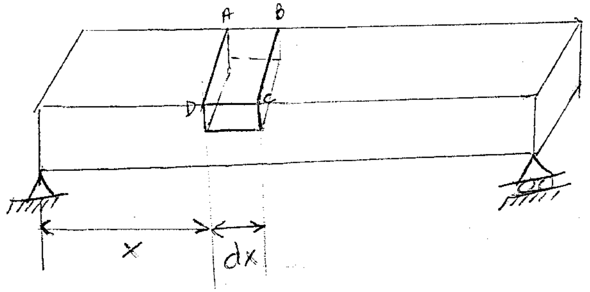
$\Sigma F_x = 0$ $\quad$ $\Sigma M = 0$ $\quad$ $\Sigma F_y = 0$

$\therefore$ τ_{yx} = horizontal shearing stress

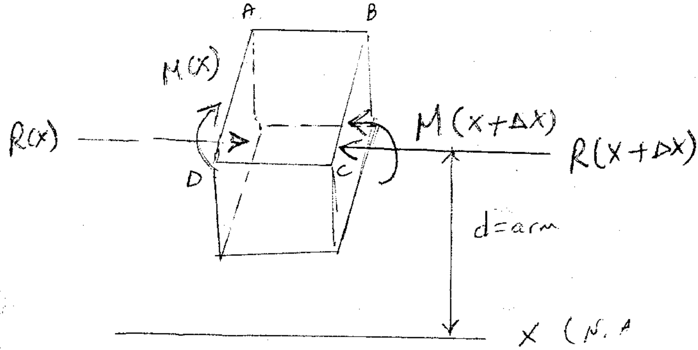
τ_{xy} = transverse shearing stress (vertical shearing stress at the cross section)

$$\underline{\tau_{xy} = \tau_{yx}}$$

Shear stresses in Beams



Note:
shear stress τ at a given cross section is due to shear force V. which will not exist unless moment is changing (ie $\frac{dM}{dx} \neq 0$).



$$R(x) = \int \sigma(x)\,dA = \int -\frac{M(x)\,y}{I}\,dA = \frac{-M(x)}{I}\int y\,dA$$

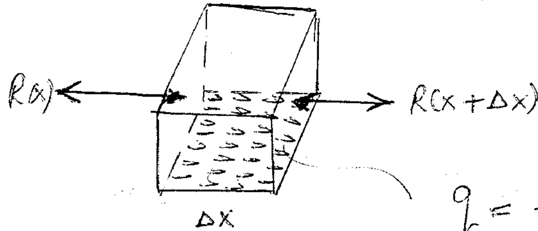
$$R(x+\Delta x) = \int \sigma(x+\Delta x)\,dA = \int -\frac{M(x+\Delta x)\,y}{I}\,dA = \frac{-M(x+\Delta x)}{I}\int y\,dA$$

Let $Q = \int y\,dA$, first moment of Inertia $= A^* \bar{y}^*$

$\therefore$ $R(x) = \frac{-M(x)}{I} Q$ $\qquad R(x+\Delta x) = \frac{-M(x+\Delta x)}{I} Q$

$$\frac{dR}{dx} = \lim_{\Delta x \to 0} \frac{R(x+\Delta x) - R(x)}{\Delta x} = \frac{-Q}{I} \lim_{\Delta x \to 0} \frac{M(x+\Delta x) - M(x)}{\Delta x}$$

$$\boxed{\frac{dR}{dx} = \frac{-VQ}{I}} \qquad \text{—— (A)}$$



q = force/unit length and constant across width (shear flow)

$\Sigma F_x = 0 \qquad -R(x) + q\,\Delta x + R(x+\Delta x) = 0$

$$\frac{R(x+\Delta x) - R(x)}{\Delta x} = -q$$

taking limit of both sides

$$\boxed{\frac{dR}{dx} = -q} \quad \text{—— (B)}$$

$$(A) = (B) \quad \Rightarrow \quad q = \frac{VQ}{I}$$

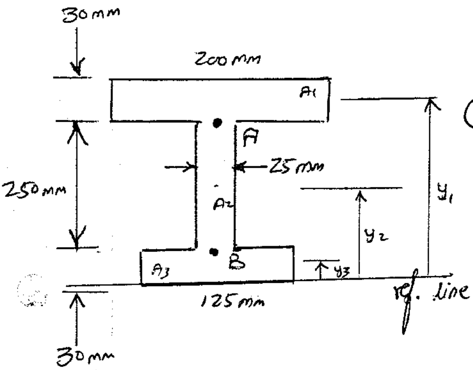
$$\Rightarrow \quad \tau = \frac{q}{t} = \frac{q}{b}$$

$$\boxed{\tau = \frac{VQ}{Ib}}$$

7-9 / 78

The beam is subjected to shear force $V = 15$ kN
Find τ_A and τ_B and show these stresses over element

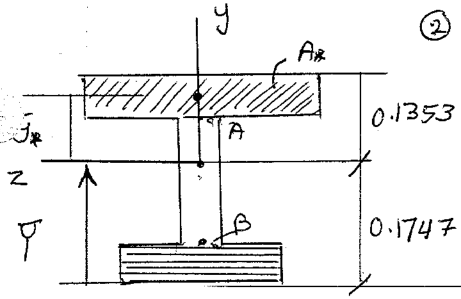
$$\boxed{Q = A_* \bar{y}_*}$$



① locate centroid and draw y-z axis through centroid.

$$\bar{Y} = \frac{\sum A_i y_i}{\sum A_i} = \frac{(200)(30)(295) + 250(25)(155) + 125(30)(15)}{(200)(30) + 250(25) + 125(30)}$$

$$= 174.7 \text{ mm}.$$



② Calculate moment of inertia about z-axis (or N.A.)

$$I_z = 0.21818 \times 10^{-3}$$

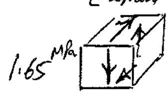
$$\tau = \frac{VQ}{Ib}$$

$$\tau_A = \frac{VQ_A}{Ib_A} = \frac{(15{,}000)\overbrace{\left(0.2 \times 0.03 \times [0.1353 - 0.015]\right)}^{Q_A}}{(0.21818 \times 10^{-3})(0.025)} = 1.99 \text{ MPa}$$

$$\tau_B = \frac{VQ_B}{Ib_B} = \frac{(15{,}000)(0.125 \times 0.03 \times [0.1747 - 0.015])}{(0.21818 \times 10^{-3})(0.025)} = 1.65 \text{ MPa}$$

Element A

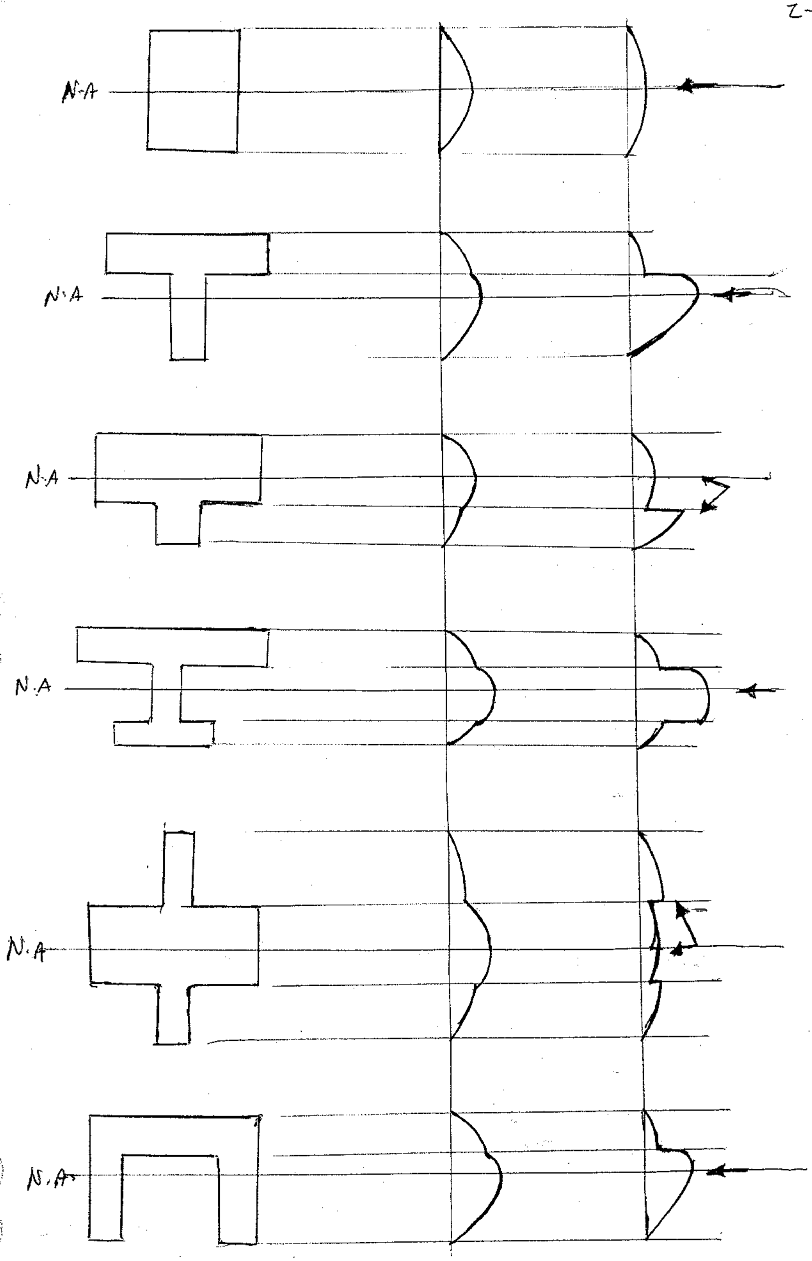
Element B

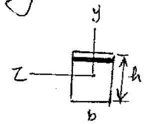


Note: arrows meet by their heads or tail.

Q.) Plot variation of Q and τ as a function of depth.

Q or q $\quad \frac{q}{b}$ or τ





$$Q = \int y\,dA = \int_{-h/2}^{y} y\,b\,dy = b\int_{-h/2}^{y} y\,dy$$

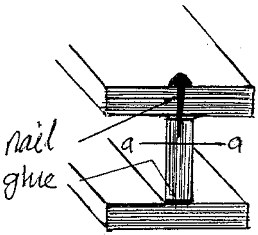
$$= \frac{b}{2}\left[y^2\right]_{-h/2}^{y}$$

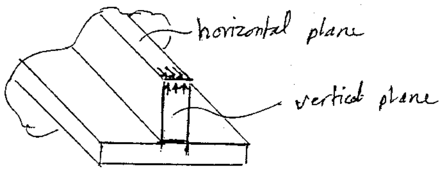
$$Q = \frac{b}{2}\left[y^2 - \left(\frac{h}{2}\right)^2\right]$$

← possible τ^{max} location

Remark

Always think about the internal vertical shear force V_y as a source for creating shear stress (indirectly through the change of moment) in two $\perp$ planes; one is the vertical plane where V_y acts and the other is the horizontal plane to keep $\Sigma F_x = 0$. These two shear stresses are equal at their intersection as shown below.

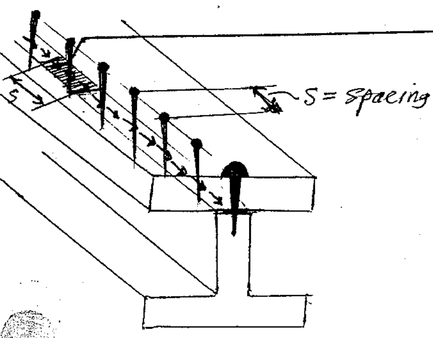




Note that for pure moment (M = constant), $V = 0$ accordingly the shear stresses in vertical and horizonal planes are zeros. and therfore no slipping and no need for nails or glue.

Objectives

① How to find the value of q or τ at any horizontal level.
② How to find shear stress at glue and force carried by nail
③ How to find the maximum shear stress due to V at a given cross section.



$$\boxed{\text{Force carried by nail} = q \cdot s}$$

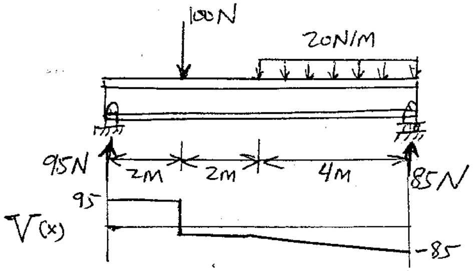
$$q = \frac{VQ}{I} = \left(\frac{V}{I}\right)Q$$

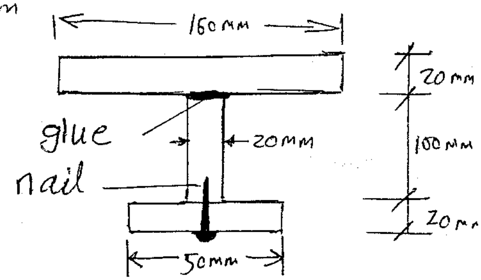
$$\tau = \left(\frac{V}{I}\right)\left(\frac{Q}{b}\right)$$

$$\tau^{max} = \left(\frac{V}{I}\right)\left(\frac{Q}{b}\right)^{Max}$$

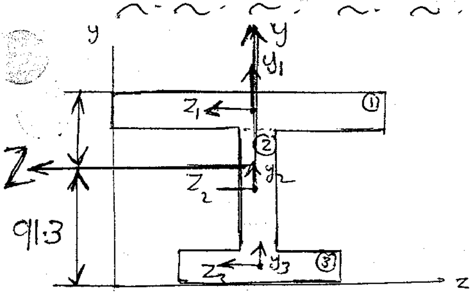
Question: For the beam shown below, find the following:

1. Glue strength (shear stress at glue level)
2. The required nail strength if spacing of nail is 50mm
3. The maximum shear stress.
4. Shear stress distribution





Finding Centroid and moment of inertia



$$\bar{Z} = 80 \text{ mm}$$

$$\bar{Y} = \frac{\Sigma A_i y_i}{\Sigma A_i}$$

$$\bar{Y} = \frac{(160)(20)(130) + 100 \times 20 \times 70 + 20 \times 50 \times 10}{160 \times 20 + 100 \times 20 + 20 \times 50}$$

$$\bar{Y} = 91.3 \text{ mm}$$

Shape	I_{z_i}	A_i	d_i	$I_{z_i} + A d_i^2$
	$\frac{1}{12}(20)^3(160)$	160×20	38.7	4.9×10^6
	$\frac{1}{12}(100)^3(20)$	100×20	21.3	2.57×10^6
	$\frac{1}{12}(20)^3\, 50$	50×20	81.3	6.64×10^6

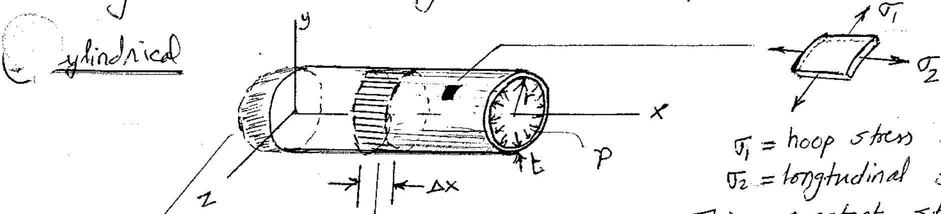
$I_Z = 14.1 \times 10^6 \text{ mm}^4$ = the sum of last col.

$I_Z = 14.1 \times 10^{-6} \text{ m}^4$

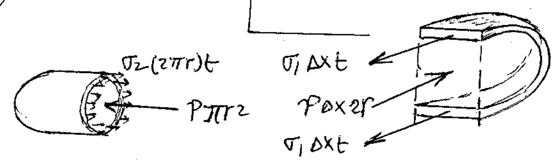
6.9 Stresses in Thin-Walled Pressure Vessels

* Analysis is limited to cylindrical and spherical vessels.

Cylindrical



σ_1 = hoop stress
σ_2 = longitudinal stress
Thin = constant stress across thickness.



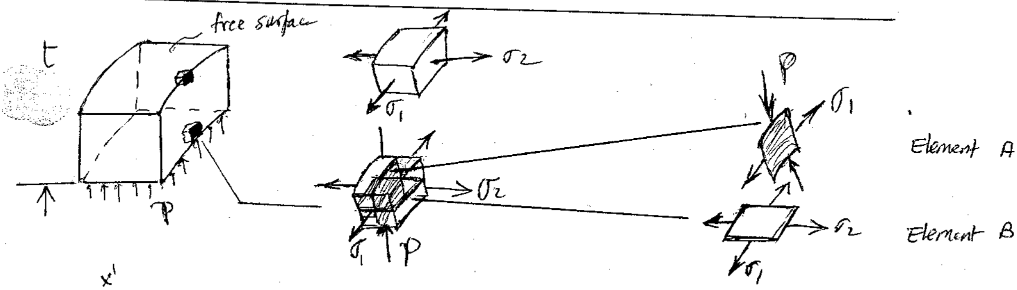
$\Sigma F_z = 0$:

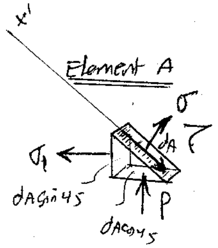
$-2r\Delta x p + 2\sigma_1 \Delta x t = 0$

$\Rightarrow \sigma_1 = \frac{pr}{t}$

$\Sigma F_x = 0$: $2\pi r t \sigma_2 - p\pi r^2 = 0$

$\sigma_2 = \frac{pr}{2t}$





$\Sigma F_{x'} = 0$: $[\sigma_1 \, dA \cos 45]\cos 45 + [p \, dA \cos 45]\cos 45 - \tau dA = 0$

$\frac{1}{2}\sigma_1 + \frac{1}{2}p = \tau$

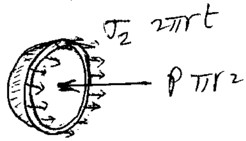
$\frac{1}{2}\left[\frac{pr}{t} + \frac{pr}{t}\left(\frac{t}{r}\right)\right] = \tau$

$\frac{pr}{2t}\left[1 + \frac{t}{r}\right] = \tau$ for $t \ll r$ $\tau^{max} = \frac{pr}{2t}$

SUMMARY : For cylindrical vessel subjected to internal pressure p of radius r and thickness t, the following stresses will be developed :

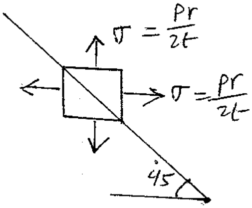
$\boxed{\sigma_1 = \text{hoop stress} = \frac{pr}{t}}$, $\boxed{\sigma_2 = \text{longitudinal} = \frac{pr}{2t}}$, $\boxed{\tau^{Max} = \frac{pr}{2t}}$

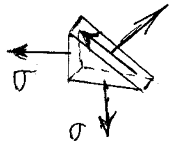
(II) Spherical

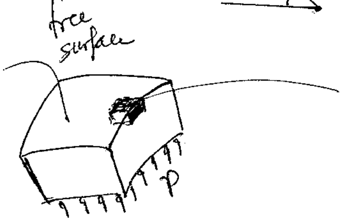


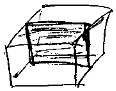
$\Sigma F_x = 0$:

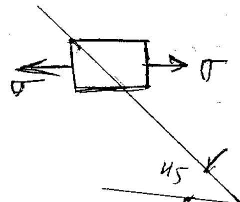
$-p\pi r^2 + \sigma_2 \, 2\pi r t = 0 \Rightarrow \sigma_2 = \frac{pr}{2t} = \sigma_1$











$\tau^{max} = \frac{1}{2}\sigma = \frac{pr}{4t}$

<u>SUMMARY</u>: For a sphere with radius r and thickness t the stresses which will develope due to internal pressure P are:

$\boxed{\sigma_1 = \sigma_2 = \frac{pr}{2t}}$ $\boxed{\tau^{max} = \frac{pr}{4t}}$

6-98 / 381

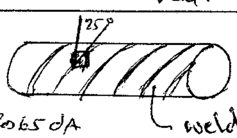
Data: $r = 250$ mm, $t = 6$ mm, $\sigma_{ult} = 400$ MPa, $P = 5.5$ MPa

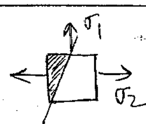
Find F.S.

$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(5.5)(250)}{2(6)} = 114.58$

$F.S. = \frac{\sigma_{ult}}{\sigma_{cal.}} = \frac{400}{114.58} = 3.49$

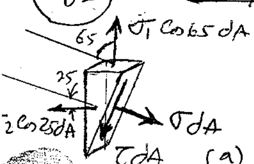
6-104 / 382





$r = 295$ mm
$t = 5$ mm
$p = 4$ MPa

Find σ and τ at weld



$\sigma_1 = \frac{pr}{t} = 236$ MPa

$\sigma_2 = \frac{pr}{2t} = 118$ MPa

(a) $\sigma dA = (\sigma_1 \cos 65\, dA)\cos 65 + (\sigma_2 \cos 25\, dA)\cos 25 \Rightarrow \sigma = 139$ MPa

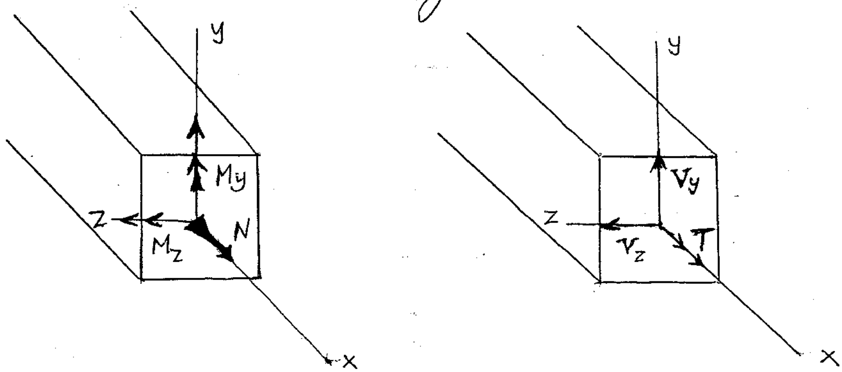
(b) $\tau dA = (\sigma_1 \cos 65\, dA)\sin 65 + (\sigma_2 \cos 25\, dA)\sin 25 \Rightarrow \tau = 45.2$ MPa

Compound stresses

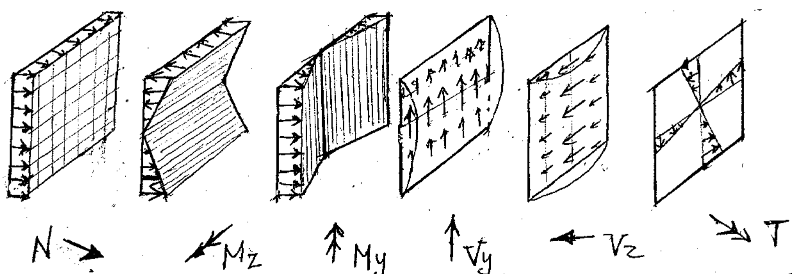
compound normal stress: Normal stresses caused by different forces and moment such as (N, M_y, M_z)

compound shear stress: Shear stresses caused by different forces and moments such as (V_y, V_z, T)

See Figure below.



$$\underbrace{\sigma = \sigma(N, M_y, M_z)}_{\text{compound normal stress}} \qquad \underbrace{\tau = \tau(V_y, V_z, T)}_{\text{compound shear stress}}$$

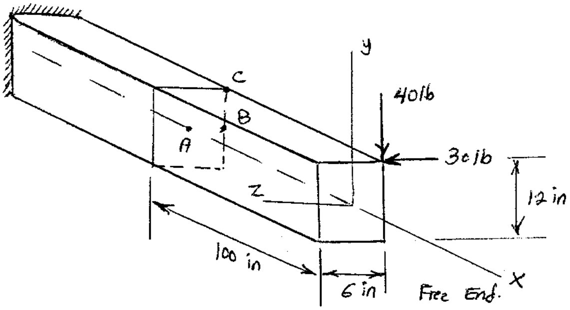


$$\boxed{\sigma = \pm \frac{N}{A} \pm \frac{M_z y}{I_z} \pm \frac{M_y z}{I_y}}$$

Select + sign whenever moment is causing tension in +ve y or z-axis

$$\tau = \begin{cases} \dfrac{V_y Q_z}{I_z b_z} & \uparrow\downarrow \\[2ex] \dfrac{V_z Q_y}{I_y b_y} & \leftrightarrows \\[2ex] T/G a b^2 & \uparrow\downarrow \rightleftarrows \end{cases}$$

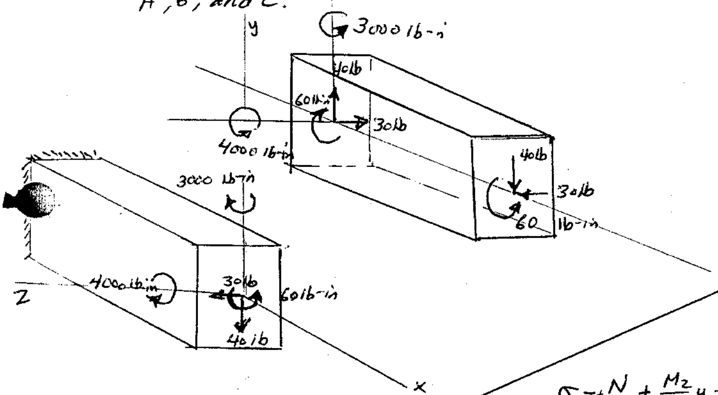
Problem



Q.) Find the state of stress at points A, B, and C which are contained in a plane 100 in from the free end

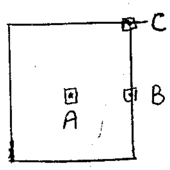
[1] Find internal forces and moments at the plane containing points A, B, and C.

$$\boxed{\begin{aligned}\sigma &= \pm \frac{N}{A} \pm \frac{M_z}{I_z} y \pm \frac{M_y}{I_y} z \\ \tau &= \frac{V_z Q_y}{I_y t_y} + \frac{V_y Q_z}{I_z t_z} + \tau_{M_x}\end{aligned}}$$



$$\boxed{\begin{aligned} &N = 0 \\ &M_z = 4000 \text{ lb-in} \\ &M_y = 3000 \text{ lb-in} \\ &V_z = 30 \text{ lb} \leftarrow \\ &V_y = 40 \text{ lb} \downarrow \\ &M_x = 60 \text{ lb-in}\end{aligned}}$$

Internal Forces and moments.

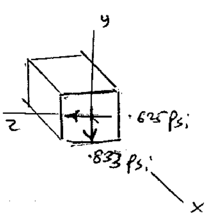


Stress at point A

$$\sigma_A = \pm \frac{N}{A} \pm \frac{M_z}{I_z} y \pm \frac{M_y}{I_y} z = 0 \quad \left\{ \begin{array}{l} y = 0 \\ z = 0 \\ N = 0 \end{array} \right\}$$

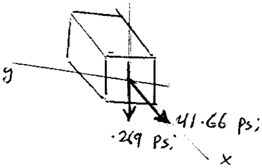
$$\tau_A = \frac{40\,[6\,(6\times3)]}{\frac{1}{12}\,(12^3\times6\times6)} \downarrow + \frac{30\,[12(3)(1.5)]}{\frac{1}{12}\,(6^3\times12)12} + 0$$

$$\tau_A = .833 \downarrow + .625 \leftarrow \text{ Psi}$$



$$\sigma_A = \begin{bmatrix} 0 & -.833 & .625 \\ -.833 & 0 & 0 \\ .625 & 0 & 0 \end{bmatrix}$$

Point B $(y=0,\ z=-3\text{ in})$



$$\sigma_B = -\frac{M_y}{I_y} z = \frac{-3000(-3)}{\frac{1}{12}(6^3(12))} = 41.66 \text{ psi}$$

$$\tau_B = \frac{V_z Q_y}{I_y t_y} + \frac{V_y Q_z}{I_z t_z} + \tau_{M_x}$$

$$= 0 + .833 \text{ psi}\downarrow + \frac{M_x}{\alpha(2b)(2a)^2}$$

$$= .833 \text{ psi}\downarrow + \frac{60}{(.246)(2\times 6)(2\times 3)^2}\uparrow \text{ psi}$$

$$= .269 \text{ psi}\downarrow$$

$$\sigma_B = \begin{bmatrix} 41.66 & -.269 & 0 \\ -.269 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Point C $(y=6,\ z=-3)$

$$\sigma_c = \frac{4000(6)}{\frac{1}{12}(12^3)(6)} - \frac{3000(-3)}{\frac{1}{12}(6^3(12))} = 69.4 \text{ psi}$$

$$\tau_c = 0 + 0 + 0 = 0$$ { because $Q_y = 0$, $Q_z = 0$ at the corner and shear stress due to M_x is also zero at corner }

$$\sigma_c = \begin{bmatrix} 69.4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

when the stress components are acting only in one plane (e.g. x-y plane, y-z plane, or x-z plane) then we call that state of stress is a <u>Biaxial State of Stress</u>.

∴ Stress state at point A is not a biaxial state of stress, whereas the stress state at points B and C are biaxial state of stress. Therefore we may express the state of stress at these two points as

$$\sigma_B = \begin{array}{c} x \\ y \end{array}\begin{bmatrix} 41.68 & -.269 \\ -.269 & 0 \end{bmatrix} \text{psi} \qquad \sigma_B = \begin{array}{c} x \\ y \end{array}\begin{bmatrix} 69.4 & 0 \\ 0 & 0 \end{bmatrix} \text{psi}$$