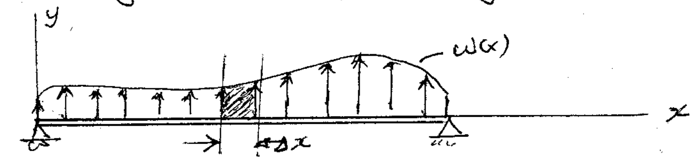
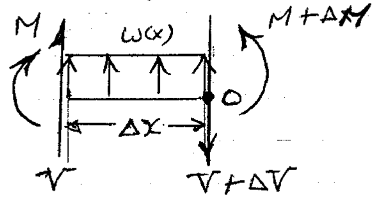
Chapter 5.

SFD & BMD and Bending Stress

Shear & Bending Moment Diagram Using Summation Method



$w(x)$ – load as a function of x.



$\Sigma F_y = 0;\quad V + w(x)\,\Delta x - (V+\Delta V) = 0$

$$\Delta V = w(x)\,\Delta x$$

$$\lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = w(x) \Rightarrow \boxed{\frac{dV}{dx} = w(x)} \quad -(1)$$

$\Sigma M_{@o} = 0 \; +\!\circlearrowleft \quad M + \Delta M - w(x)(\Delta x)\left(\frac{\Delta x}{2}\right) - V\,\Delta x - M = 0$

$\cancel{M} + \Delta M - V\,\Delta x - \cancel{M} = 0$ (load term is higher o.t.

$$\lim_{\Delta x \to 0} \frac{\Delta M}{\Delta x} = V(x) \Rightarrow \boxed{\frac{dM}{dx} = V(x)} \quad -(2)$$

Integrate (1) $\displaystyle\int_{V_A}^{V_B} dV = \int_{x_A}^{x_B} w(x)\,dx \Rightarrow V_B - V_A = \int_{x_A}^{x_B} w(x)\,dx =$ area of load.

$\therefore$ $\boxed{V_B = V_A + \text{area of the loading function between A \& B}}$

Integrate (2) $\displaystyle\int_{M_A}^{M_B} dM = \int_{x_A}^{x_B} V(x)\,dx \Rightarrow M_B - M_A = \int_{x_A}^{x_B} V(x)\,dx =$ area of shear.

$\boxed{M_B = M_A + \text{area of the shear function between A \& B}}$

Notes To Remember When Drawing SFD and BMD.

① The change of shear between two points A and B is equal to the area of the loading between A and B. OR (The shear at point B is equal to the shear at point A + the area of load between A and B).

② The change of moment between two points A and B is equal to the area of shear diagram between A and B. OR. (The moment at point B is equal to the moment at point A + the area of shear between A and B).

③ The shear is changed by the magnitude of the Concentrated Load. Upward load increase shear by its magnitude whereas downward load decrease shear by its magnitude.

④ Concentrated moment does not effect the value of shear but increases moment by its magnitude if the applied moment is clockwise and decrease moment by its magnitude if it is applied counterclockwise.

⑤ The moment is maximum wherever shear changes its sign. Both magnitude and location of max. moment are important to be shown on the bending moment diagram.

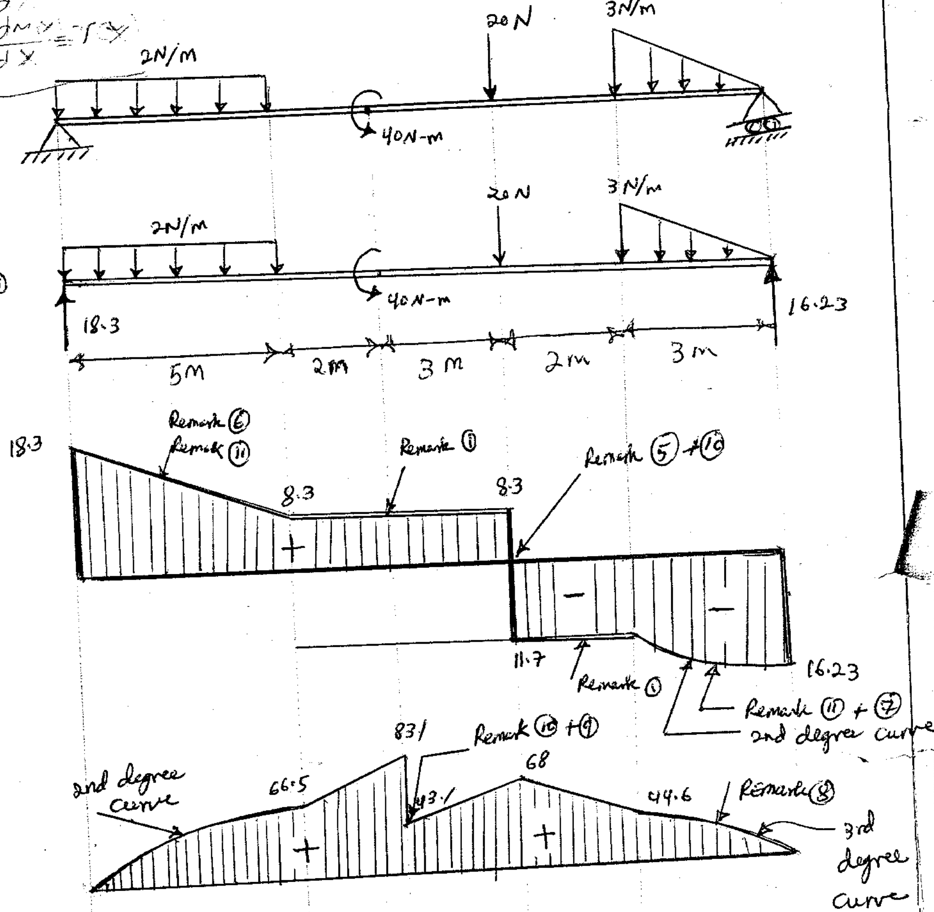
$$\frac{dV(x)}{dx} = \omega(x) \quad \text{———} \quad V_B = V_A + \int_{x_A}^{x_B} \omega(x)\,dx$$

$$\frac{dM(x)}{dx} = V(x) \qquad M_B = M_A + \int_{x_A}^{x_B} V(x)\,dx$$

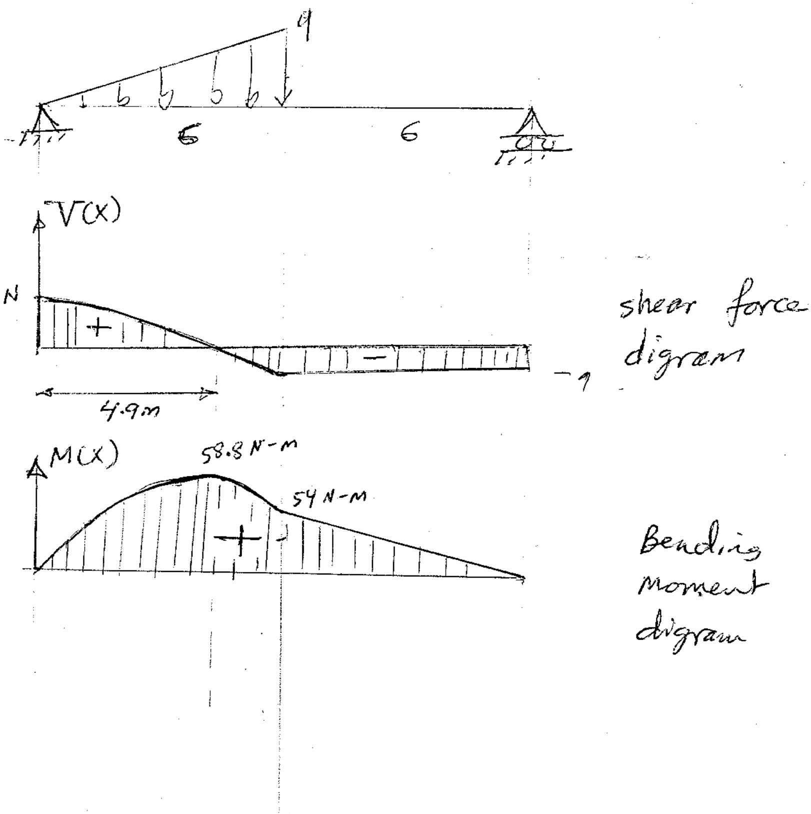
$$\frac{dV(x)}{dx} = w(x)$$

$$\frac{dM(x)}{dx} = -V(x)$$

STEP(1)

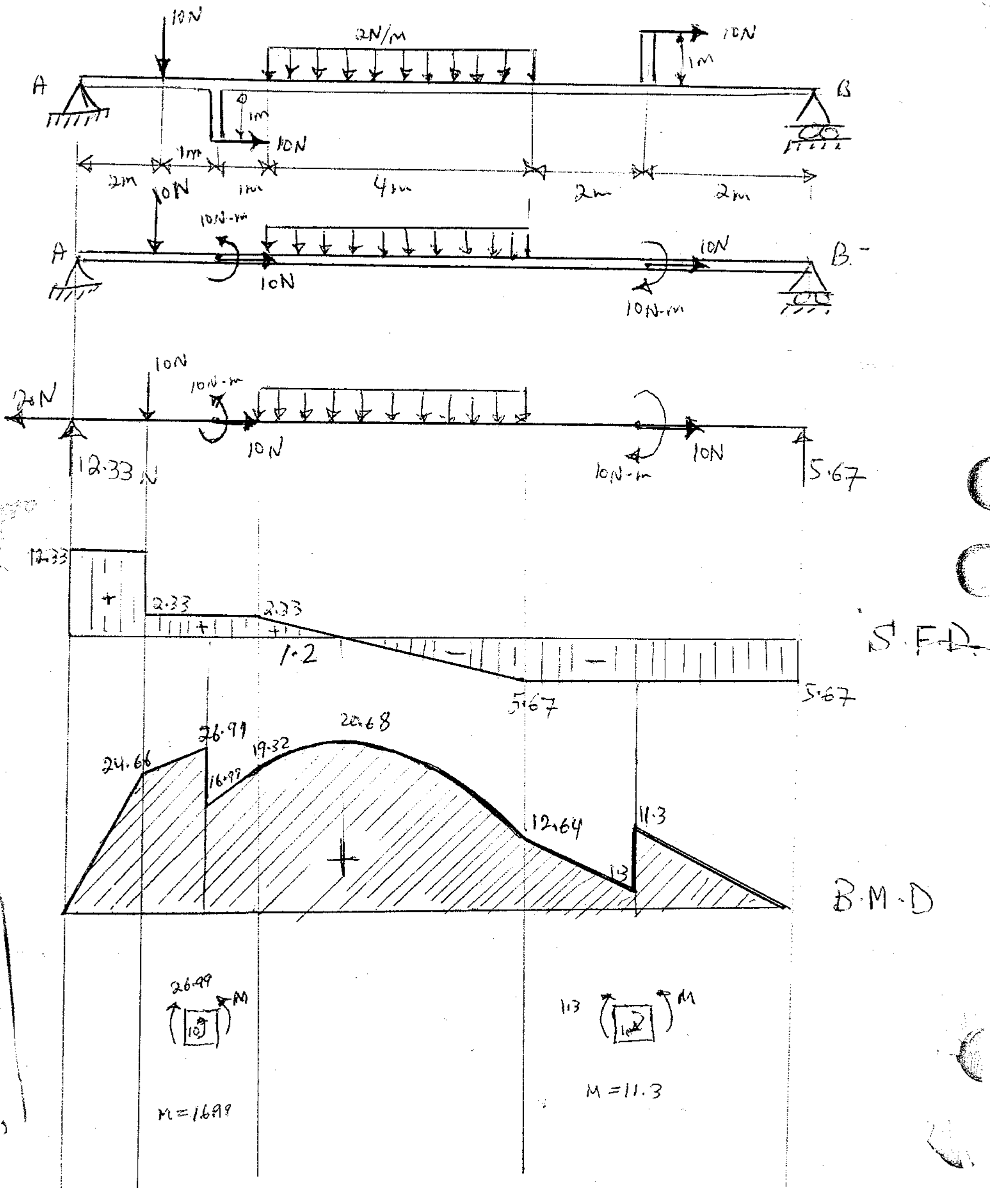


(3) Drawing V and M:



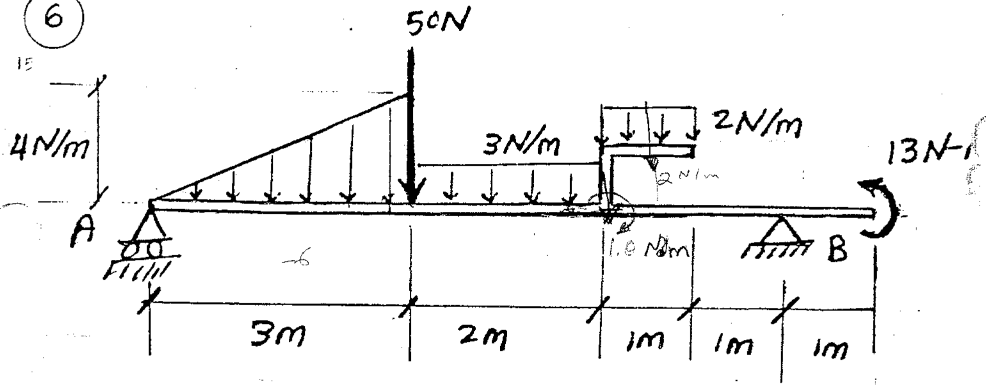
<u>Remark</u>: points of zero shear correspond to maximum moments

* locations of zero shear and maximum moment are important to indicate on the digram


10N
2N/m
10N
1m
A
B
1m
10N
2m
1m
1m
4m
2m
2m
10N
10N-m
10N
10N
10N-m
20N
10N
10N-m
10N
10N
10N-m
12.33 N
5.67
12.33
2.33
2.33
1.2
5.67
5.67
S.F.D
26.99
20.68
24.66
19.32
16.99
12.64
11.3
13
B.M.D
26.99
M
10
M=16.98
13
M
10
M = 11.3

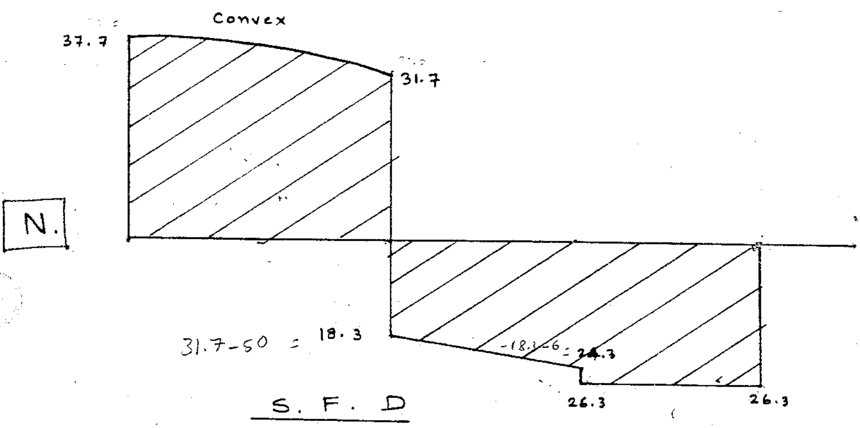
CE 203 H.W# 7.

6



$R_A = 37.7N$

$R_B = 26.3N$



S. F. D

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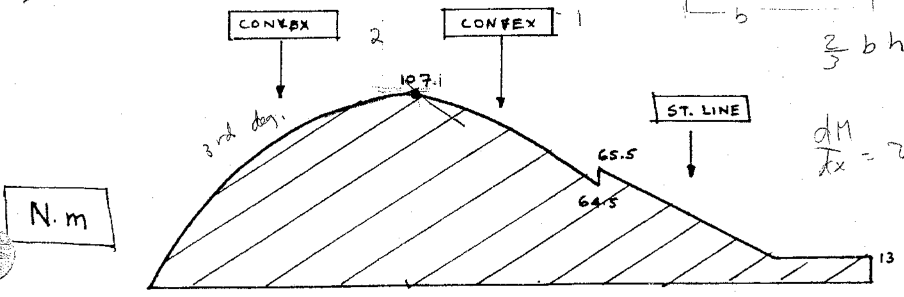
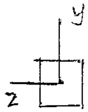


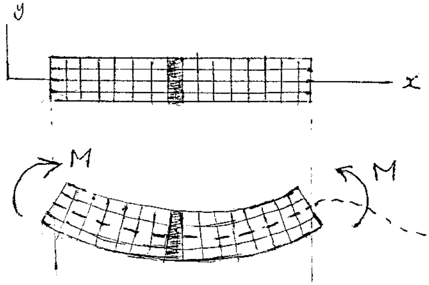
$\frac{2}{3}bh$

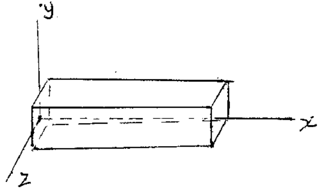
$\frac{dM}{dx} = v(x)$

B. M. D

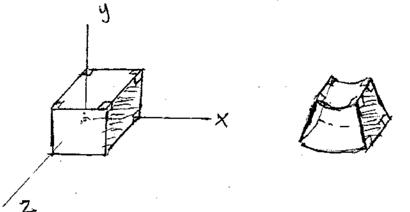
Bending stresses







Neutral axis (N.A)



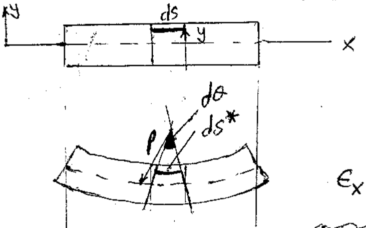
Assumption:
Planes ⊥ to central line before bending remain ⊥ to center line after bending

1. No change in the 90° angle of x-y plane (i.e $\gamma_{xy}=0 \Rightarrow \tau_{xy}=0$)
2. No change in the 90° angle of x-z plane (i.e $\gamma_{xz}=0 \Rightarrow \tau_{xz}=0$)
3. No change in the 90° angle of y-z plane (i.e $\gamma_{yz}=0 \Rightarrow \tau_{yz}=0$)

In general at any particular element of the vertical plane there are two components of shear and one component of normal stress (i.e $\tau_{xz}, \tau_{xy}, \sigma_x$). But shear stresses are zeros, therefore there is only one component of stress and that is normal stress σ_x.

The objective is to find relationship between internal moment M and the associated normal stress $\sigma_x \Rightarrow \sigma_x = \sigma_x(M)$?

① Finding ϵ_x:



ds = original length
ds* = deformed length

$ds = \rho\, d\theta$

$ds^* = (\rho - y)\, d\theta$

$$\epsilon_x = \frac{ds - ds^*}{ds} = \frac{(\rho-y)d\theta - \rho d\theta}{\rho d\theta}$$

$$\boxed{\epsilon_x = \frac{-y}{\rho}}$$

1. strain proportional to distance from N.A.
2. for $y > 0$ ($\epsilon_x < 0$) — compression
3. for $y < 0$ ($\epsilon_x > 0$) — tension

Flexure Formula

) Kinematics : $\epsilon_x = \dfrac{-y}{\rho}$ ——— ①✓

② Equilibrium: $M_z = -\int y\sigma_x \, dA$ ——— ②✓

③ Material Behavior : $\sigma_x = E\epsilon_x$ ——— ③✓

Using ③ into ① : $\dfrac{\sigma_x}{E} = \dfrac{-y}{\rho} \Rightarrow \underline{\underline{\dfrac{1}{\rho} = \dfrac{-\sigma_x}{Ey}}}$ — Ⓐ

Using ① and ③ into ② ;

$$M_z = -\int -y\,\sigma_x \, dA$$

$$= -\int -y\,E\epsilon_x \, dA$$

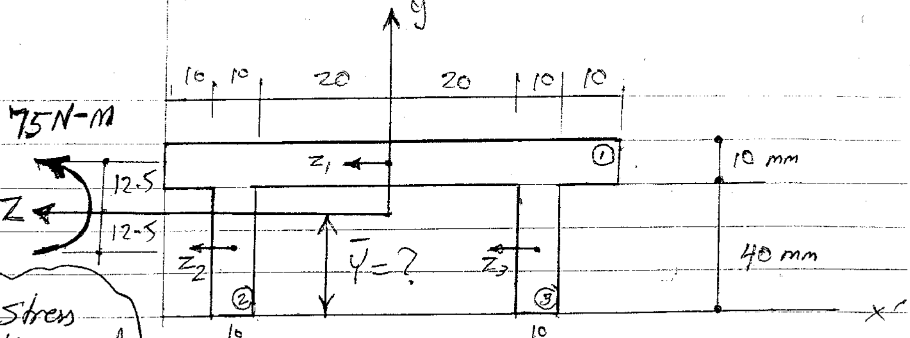
$$= -\int y\,E\left(\frac{-y}{\rho}\,dA\right)$$

$$= \frac{E}{\rho}\int y^2 \, dA$$

$$M_z = \frac{E}{\rho} I_z \Rightarrow \underline{\underline{\frac{1}{\rho} = \frac{M_z}{EI_z}}} \text{ — Ⓑ}$$

Upon comparing Ⓐ and Ⓑ, one may write

$$\boxed{\sigma_x(x,y) = \frac{-M_z(x)\;y}{I_z}}$$



Find stress distribution and σ_{max}

Step ①: Find centroidal axis (Z-axis)

Area	A_i	y_i	$A_i y_i$
	800	45	36000
	400	20	8000
	400	20	8000
	1600		52000

$$\bar{Y} = \frac{\sum A_i y_i}{\sum A_i} = \frac{52000}{1600} = 32.5 \text{ mm}$$

Step ②: Find moment of inertia about Z-axis

A_i	I_{zi}	d_i	Ad_i^2
	$\frac{1}{12}(10)^3 80$	12.5	$800(12.5)^2$
	$\frac{1}{12}(40)^3(10)$	12.5	$400(12.5)^2$
	$\frac{1}{12}(40)^3(10)$	12.5	$400(12.5)^2$
	113333		250000

$$I_z = \sum(I_{zi} + Ad_i^2) = 113333 + 250000$$
$$= 363333 \text{ mm}^4$$

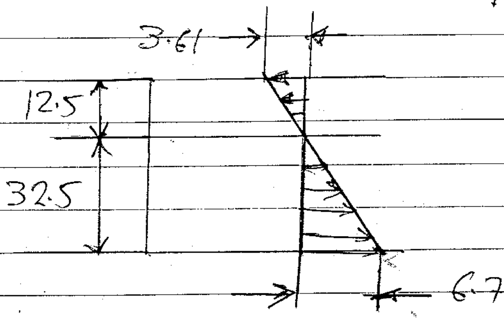
$$\sigma = \frac{-M_z y}{I_z} = \frac{-75(1000)(y)}{363333}$$

$$\sigma_t = \frac{-75(1000)\, y_t}{363333} = \frac{-75(1000)(+12.5)}{363333}$$

$$= -3.61 \text{ MPa}$$

$$\sigma_b = \frac{-75(1000)\, y_b}{363333} = \frac{-75(1000)(-32.5)}{363333}$$

$$= +6.7 \text{ MPa}$$



You can also find 6.7 MPa from similar traingle

$$\frac{3.61}{12.5} = \frac{\sigma_b}{32.5} \Rightarrow \sigma_b = 6.7 \text{ MPa.}$$

Normal Stresses Due to Bending in Beams

The objective is to know how to find the following:

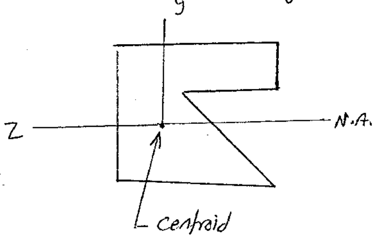
(a) Normal stress at any point of the cross section

(b) stress distribution within cross section

(c) stress resultant acting over any part of the cross section

(d) The max. load which can be carried by the beam.

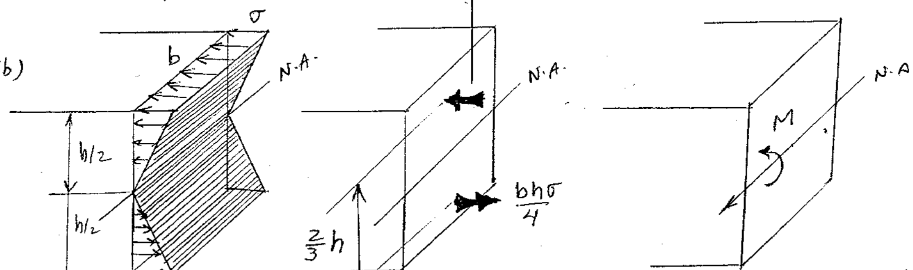
(a) To find the stress due to moment of any point in the cross section, you need to find its $\underline{\underline{y}}$. (distance from N.A)



$\frac{1}{2}\left(\frac{h}{2}\sigma\right)b$ = volume of prism

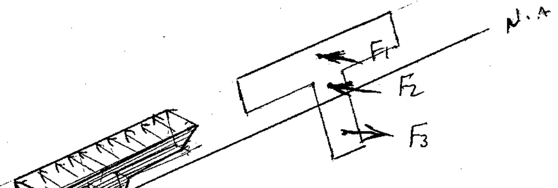
$\frac{1}{2}\sigma\left(\frac{bh}{2}\right)$ = average stress X area

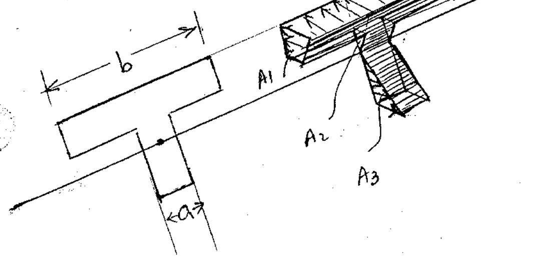
(b)



$$M = \frac{bh\sigma}{4}\left(\frac{2}{3}h\right) = \frac{2bh^2\sigma}{12}$$

(c) stress resultant





$F_1 = A_1 b$ = volume of trapezoidal prism

$F_2 = A_2 a$ = volume of triangular prism

$F_3 = A_3 a$ = volume of triangular prism