Chapter 4.

Torsion

TORSION

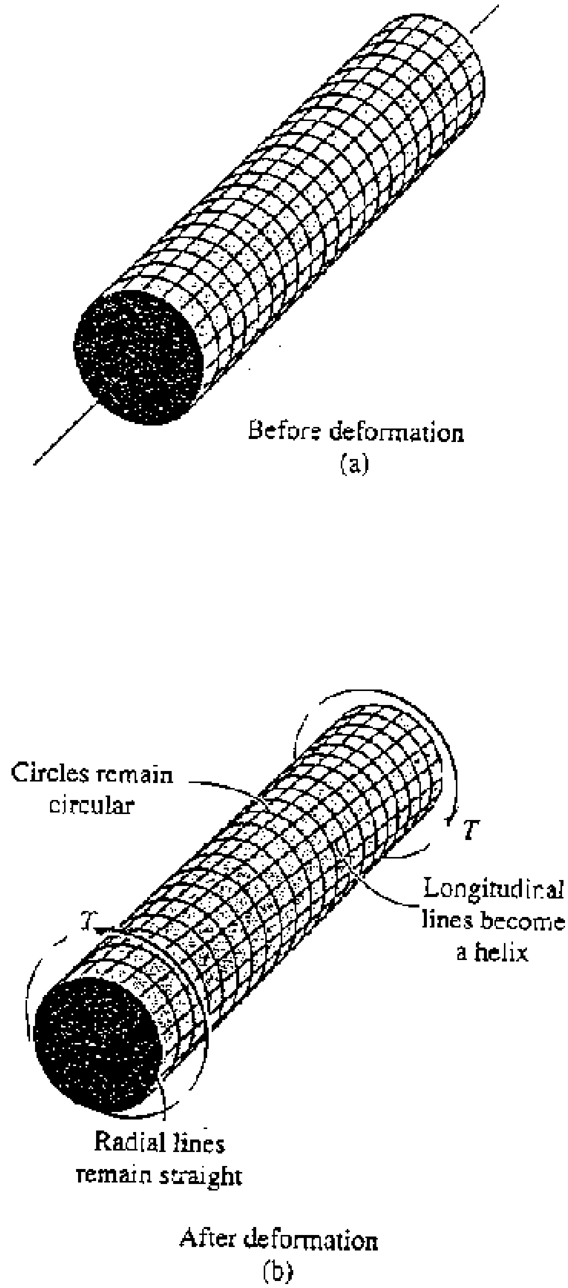
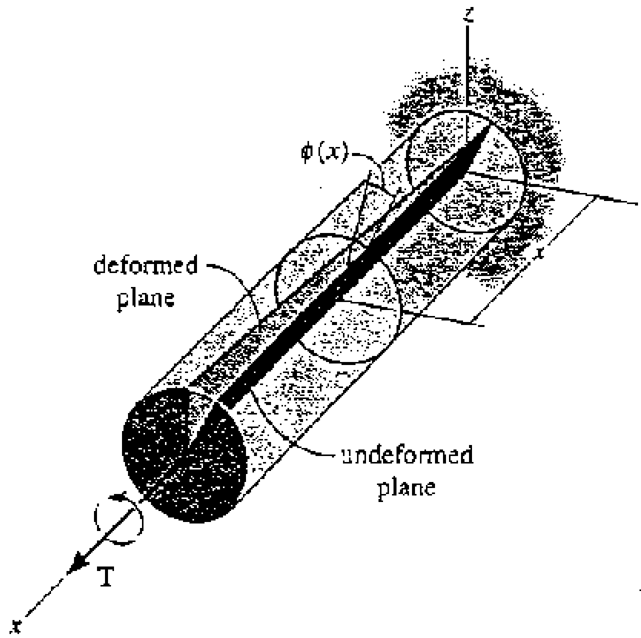


Fig. 5-1



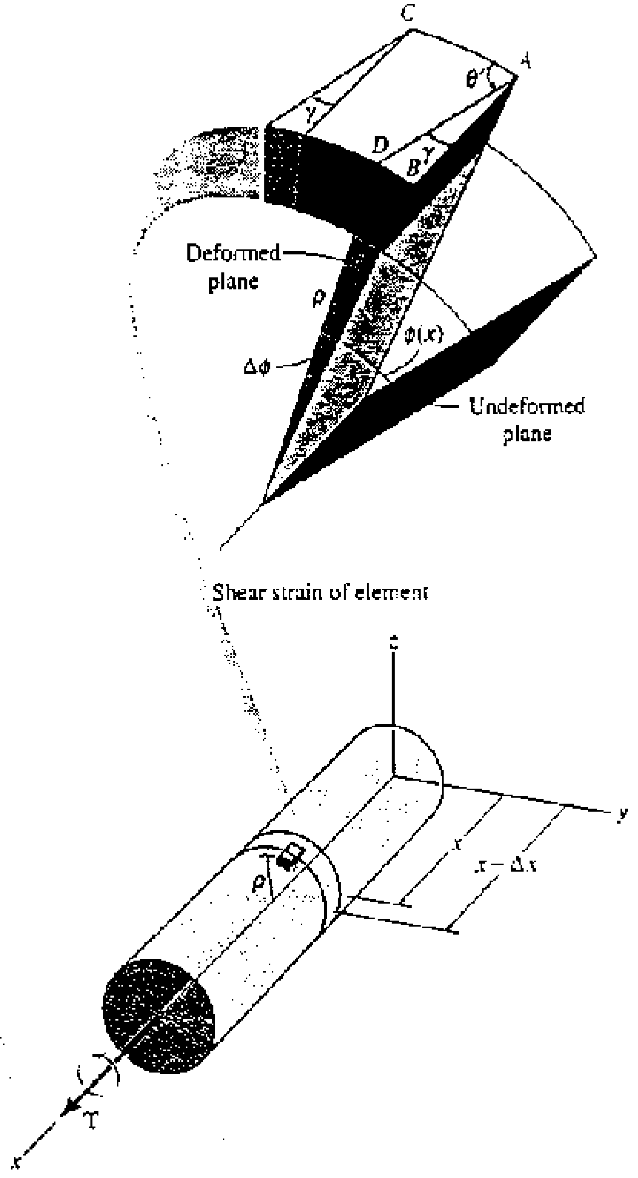
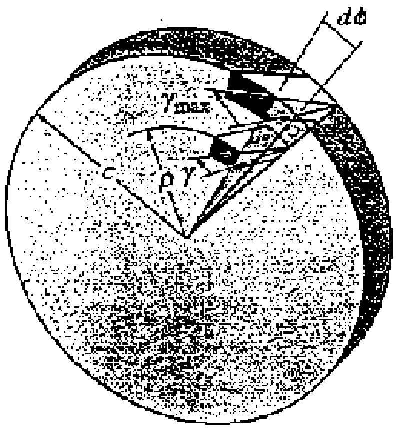


Fig. 5–3

$$BD = \rho \, d\phi = dx \, \gamma$$

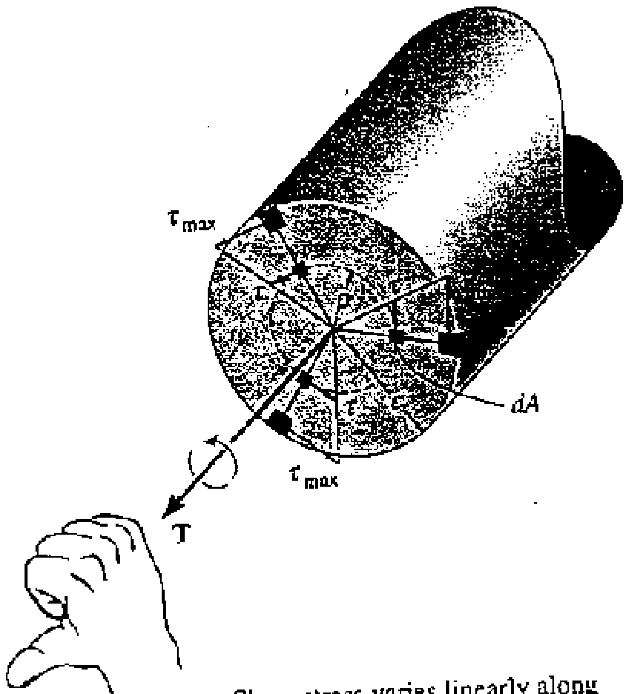
$$\gamma = \rho \frac{d\phi}{dx}$$

$$\gamma = \left(\frac{\rho}{c}\right)\gamma_{max}$$



The shear strain for the material increases linearly with ρ, i.e., $\gamma = (\rho/c)\gamma_{max}$.

Fig. 5–4



Shear stress varies linearly along each radial line of the cross section.

Fig. 5–5

$$\tau = \left(\frac{\rho}{c}\right)\tau_{max}$$

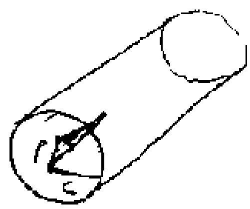
$$\boxed{\tau_{max} = \frac{Tc}{J}}$$

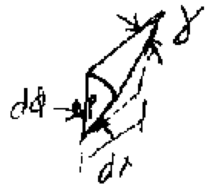
$$\boxed{J = \frac{\pi}{2}c^4}$$

$$\boxed{\tau = \frac{T\rho}{J}}$$

$$\boxed{J = \frac{\pi}{2}(c_o^4 - c_i^4)}$$

Shear stress formula & Angle of twist

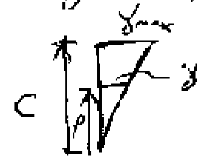




$$\rho \, d\phi = \gamma \, dx$$
$$\gamma = \rho \frac{d\phi}{dx}$$

for a given x, $\frac{d\phi}{dx}$ = constant

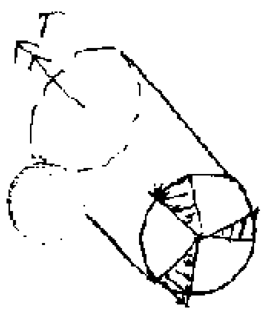
therefore γ is proportional to ρ



$$\frac{\gamma_{max}}{c} = \frac{\gamma}{\rho}$$

$$\therefore \quad \gamma = \rho \frac{\gamma_{max}}{c}, \quad \text{since } \tau \propto \gamma$$

$$\therefore \quad \tau = \rho \frac{\tau_{max}}{c}$$



$$T = \int dT = \int \rho (\tau \, dA)$$
$$= \int \rho \left(\rho \frac{\tau_{max}}{c} \right) dA$$
$$= \frac{\tau_{max}}{c} \int \rho^2 dA$$
$$T = \frac{\tau_{max}}{c} \cdot J$$

$$\therefore \quad \tau_{max} = \frac{Tc}{J} \quad \text{or} \quad \tau = \frac{T\rho}{J}$$

$J = \frac{\pi c^4}{2}$ for  c , polar moment of inertia

$J = \frac{\pi}{2}(c_o^4 - c_i^4)$  c_i

Angle of Twist ϕ

$$d\phi = \gamma \frac{dx}{\rho}$$

$$= \frac{\tau}{G}\frac{dx}{\rho}$$

$$= \frac{1}{G}\left(\frac{T\rho}{J}\right)\frac{dx}{\rho}$$

$$d\phi = \frac{T}{GJ}\,dx$$

$$\phi = \int_0^L \frac{T}{GJ}\,dx$$

For constant T, G, and J, the above can be integrated to yield.

$$\phi = \frac{TL}{GJ}$$, and for many segments,

$$\phi = \sum_{i=1}^{n} \frac{T_i L_i}{G_i J_i}$$

5.3 Power Transmission

angular velocity, (angular distance per unit time)

$W = Fd$ or $W = T\theta$, work

$P = \frac{dW}{dt}$ ($P = FV$ or $P = T\frac{d\theta}{dt} = T\omega$)

$P = T[2\pi f]$, f = frequency $\left(\frac{\#\text{ of revolution}}{\text{second}}\right)$

P = Watt ($hp = 550\ \frac{\text{lb-ft}}{s}$)

T = N-m (ft-lb)

ω = rad/s

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From table ; $G = 11.0 \times 10^3$ ksi $\qquad A = \pi r^2 \Rightarrow r = 1.6925$

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$$\tau_c = \frac{Tc}{J} = \frac{(4000)(1.692568)}{(\pi/2)(1.692568)^4} = 525 \text{ psi} \quad \text{, Circle}$$

$$\tau_r = \frac{4.81\, T}{a^3} = \frac{(4.81)(4000)}{(3)^3} = 713 \text{ psi}$$

$$\phi_c = \frac{TL}{GJ} = \frac{(4000)(36)}{(11.0\times10^6)(\pi/2)(1.692568)^4} = 1.0456 \times 10^{-3} \text{ rad}$$

$$\phi_r = \frac{7.10\, TL}{a^4 G} = \frac{(7.10)(4000)(36)}{(3^4)(11.0\times10^6)} = 1.1474 \times 10^{-3} \text{ rad}$$

∴ As a conclusion stress is smaller and angle twist is also smaller for the circular shaft compared to square one

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$G = 26 \times 10^9$ pa

$$\tau_{all} = \tau_{cal} \;:\; 125\times10^6 = \frac{T}{2A_m t} = \frac{75000}{2[4a(2a) + \pi a^2].007}$$

$\Rightarrow a = 62$ mm

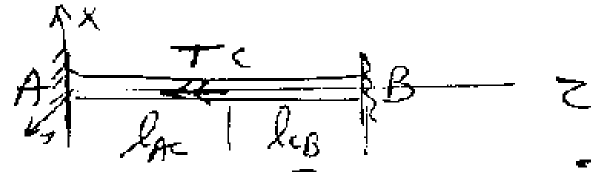
$$\phi = \frac{TL}{4GA_m^2}\frac{S}{t} = \frac{(75000)(2)(4+\pi)2a}{(4)(26\times10^9)\{8+\pi\}^2(0.007)(a^4}$$

$= 0.0994$ rad.

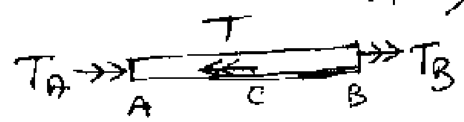
$= 5.69°$

5.5 Statically Indeterminate Members

Like statically indeterminate of axially loaded members, one needs both equations of equilibrium and compatability equations.



① Equilibrium :



$$\boxed{T_B + T_A - T = 0}$$ T is given

② Compatability : $\phi_{A/B} = 0$, $(\phi_{AC} + \phi_{CB} = 0)$, $\left(\frac{T_{AC}\, l_{AC}}{GJ} + \frac{T_{CB}\, l_{CB}}{GJ} = 0\right)$

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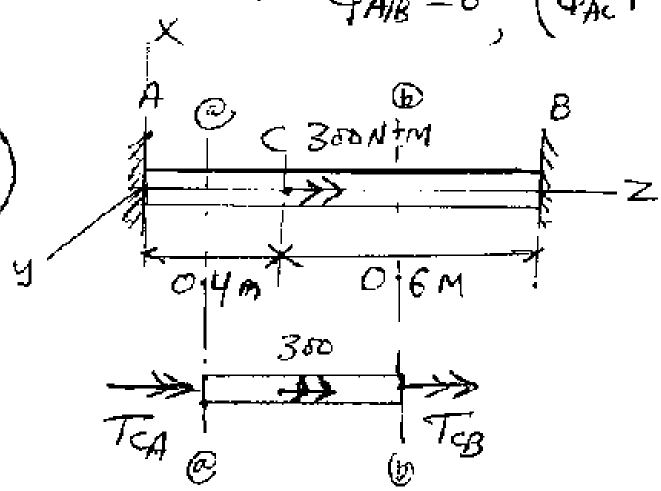


$T = (3\times10^3)\left(\frac{100}{1000}\right) = 300$ N-m

positive torque is pointing in the positive z- axis

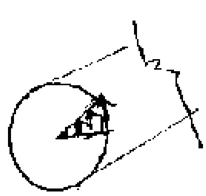
① Equilibrium : $T_{CB} + T_{CA} + 300 = 0$

② Compatability : $\phi_{A/B} = 0 \quad \left(\frac{T_{CA}\,(0.4)}{GJ} + \frac{T_{CB}\,(0.6)}{GJ} = 0\right)$

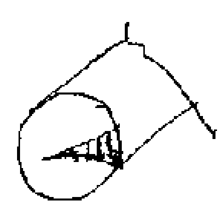
$$\therefore \quad T_{CA} + T_{CB} = -300 \quad \text{——(1)}$$
$$0.4\, T_{CA} + 0.6\, T_{CB} = 0 \quad \text{——(2)}$$

$\therefore \quad T_{CA} = -180$ N-m $\Rightarrow \tau_{AC} = \frac{Tc}{J} = \frac{-(180)(0.02)}{\left(\frac{\pi}{2}\right)(0.02)^4} = -14.3$ MPa

$T_{CB} = -120 \Rightarrow \tau_{CB} = \frac{Tc}{J} = \frac{-(120)(0.02)}{\frac{\pi}{2}(0.02)^4} = -9.55$ MPa



Section a-a



Section b-b

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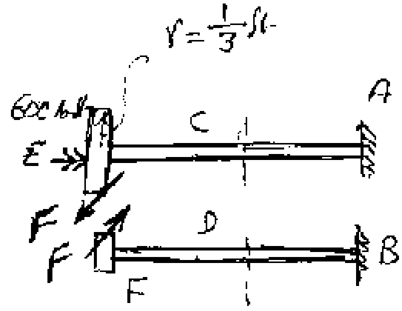
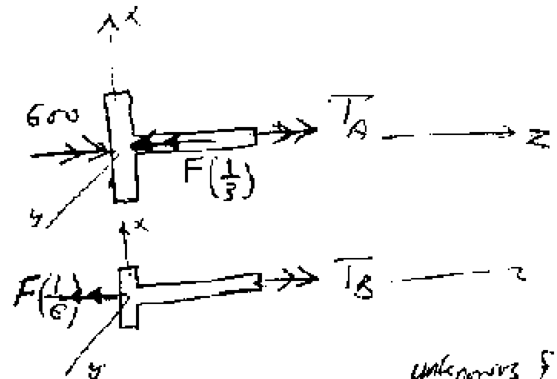




unknowns $\{T_A, T_B, F\}$

shaft A : $\quad T_A + 600 - \frac{F}{3} = 0 \quad$ —(1)

shaft B : $\quad T_B - \frac{F}{6} = 0 \quad$ —(2)

Combine (1) & (2) $\Rightarrow$ $\boxed{T_A + 2T_B + 600 = 0}$ (A) Equilibrium Eq.

Compatability Equation

$$r_E \phi_E = r_F \phi_F$$ (Same angular distance $r\phi$)

$$4\,\phi_E = 2\,\phi_F$$

$$\phi_E = \frac{1}{2}\,\phi_F$$

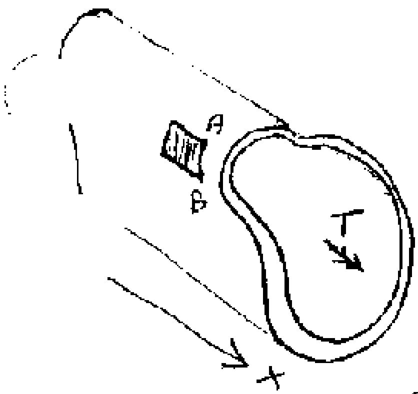
$$\frac{T_A L}{GJ} = \frac{1}{2}\,\frac{T_B L}{GJ} \Rightarrow \boxed{2T_A = T_B}$$ (B)

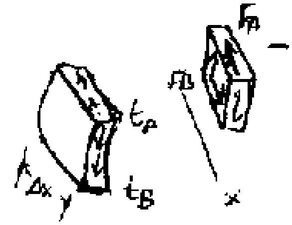
solving equations (A) & (B) $\Rightarrow$ $T_A = 120$ lb-ft
$T_B = 240$ lb-ft

$$\left(\tau_{BD}\right)^{max} = \frac{T_B\,C}{J} = \frac{(240)(12)(0.75)}{(\frac{\pi}{2})(0.75)^4} = 4.35 \text{ ksi}$$

$$\left(\tau_{AC}\right)^{max} = \frac{T_A\,C}{J} = \frac{(120)(12)(0.75)}{(\frac{\pi}{2})(0.75)^4} = 2.17 \text{ ksi}$$

5-7 Thin-Walled Tubes Having Closed Cross Sections



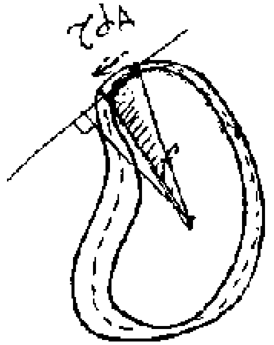


f_A — shear stress acting in all four faces meet by tails or heads.

$\Sigma F_x = 0; \quad f_A = f_B$

$\tau_A t_A \Delta x = \tau_B t_B \Delta x = \text{const.}$

$\boxed{q = \tau t} \Leftarrow \tau_A t_A = \tau_B t_B = q$ (shear flow)



$$T = \int dT = \int dF \rho = \int \tau dA \rho = \int \tau t ds \rho = \int q ds \rho$$
$$= \int q(2da) = 2q \int da = 2q a$$

$\boxed{q = T/2a}$ where T is the applied torque and a is the area enclosed by center line

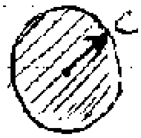
$da = \frac{1}{2} ds \rho$

$2da = \rho ds$

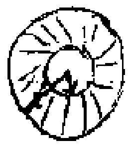
$$\boxed{\phi = \frac{TL}{4a^2 G} \int \frac{ds}{t}}$$, The angle of twist

TORSION

① Solid / Hallow Circular Sections



$\tau = \frac{T\rho}{J}$, $\tau^{max} = \frac{Tc}{J}$, $J = \frac{\pi d^4}{32}$

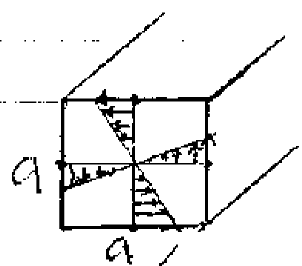


, $\tau^{max} = \frac{T c_o}{J}$; $J = \frac{\pi}{32}(d_o^4 - d_i^4)$

$$\phi = \frac{TL}{GJ}$$

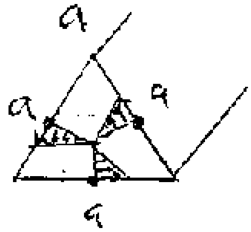
② Solid Non-Circular sections

See table 5-1 page 221

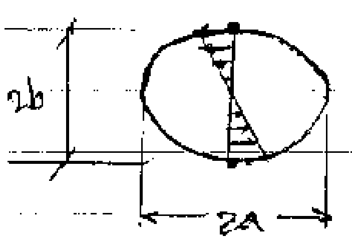


• points of $\tau^{max} = \frac{4.81T}{a^3}$

at corners $\tau = 0$, $\phi = \frac{7.10TL}{a^4 G}$



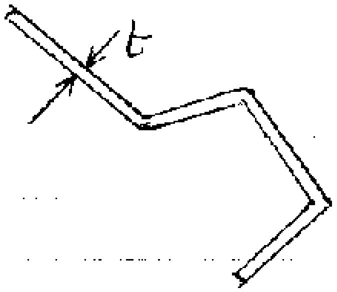
$\tau^{max} = \frac{20T}{a^3}$, $\phi = \frac{46TL}{a^4 G}$



$\tau^{max} = \frac{2T}{\pi a b^2}$, $\phi = \frac{a^2+b^2}{\pi a^3 b^3 G} TL$

$$\phi = \frac{TL}{GJ}$$

③ Thin-Walled Open Sections

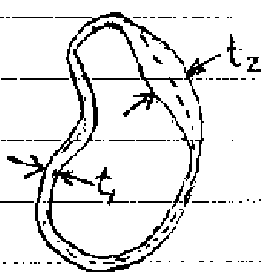


$$\tau = \frac{Tt}{J} \quad , \quad J = \frac{1}{3} L t^3$$

L = length
t = thickness

$$\phi = \frac{TL}{GJ}$$

④ Thin-Walled Closed Sections



$$\tau = \frac{q}{t} \quad , \quad q = \text{shear flow}$$

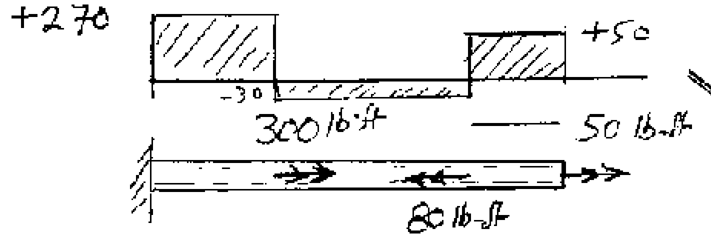
$$\tau^{max} = \frac{q}{t_{min}} \quad , \quad q = \frac{T}{2A_m}$$

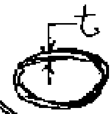
A_m = area enclosed by the central line

$$\phi = \frac{TL}{G}\left(\frac{1}{4A_m^2}\int \frac{ds}{t}\right)$$

where s = the length of central line
t = thickness

when t = constant , $\phi = \dfrac{TLs}{4GA_m^2 t}$





Thin-walled closed section

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$$T_{max} = 270 \text{ lb-ft}$$

$$\tau^{max} = \frac{q}{t} = \frac{T^{max}}{2[\pi b(0.5b)]t} = \tau_{all}$$

$$= \frac{(270)(12) \text{ lb-in}}{2\pi(b)(0.5b)(0.2} = 8.0\times10^3$$

q = shear flow

$$\Rightarrow \boxed{b = 0.803 \text{ in}}$$

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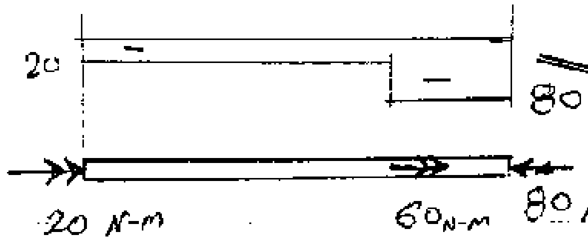
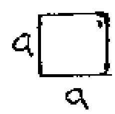




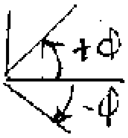
$$T^{max} = 80 \text{ N-m}$$

$G = 26$ GPa (from table)

Solid non-circular sections

$$\tau^{max} = \frac{4.81\,T^{Max}}{a^3} \quad \text{(Formula from table 5-1 page 221)}$$

$$= \frac{(4.81)(80)}{(0.025)^3} = 24.6 \text{ MPa}$$



$$\phi = \frac{7.10\,TL}{Ga^4} = \frac{(7.10)(-20)(1.5)}{(26\times10^9)(0.025)^4} + \frac{(7.10)(-80)(0.5)}{(26\times10^9)(0.025)^4} = -0.0489$$

$$= -1.4^\circ = 1.4^\circ$$

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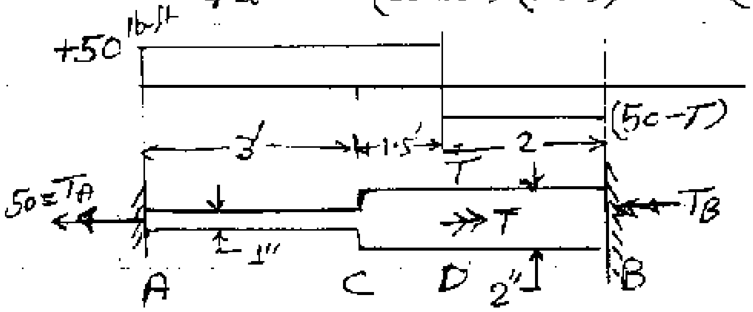

Two unknowns (T & T_B)
Need two equations
($G = 11\times10^6$ psi) From table

$J = \frac{\pi}{2}c^4$

1. Equilibrium Equation: $T - 50 - T_B = 0 \Rightarrow T_B = T - 50 = T_{DB}$

2. Compatibility Equation: $\frac{T_{AC}l_{AC}}{GJ_{AC}} + \frac{T_{CD}l_{CD}}{GJ_{CD}} + \frac{T_{DB}l_{DB}}{GJ_{DB}} = 0 \quad (\Phi_{A/B} = 0)$

$$\frac{[(50)(12)]_{\text{lb-in}}[3\times12]}{11\times10^6\,\frac{\pi}{2}(1)^4} + \frac{[(50)(12)][1.5\times12]}{(11\times10^6)(\frac{\pi}{2})(2)^4} + \frac{(50-T)(12)(2\times12)}{(11\times10^6)(\frac{\pi}{2}(2)^4)} = 0$$

solving for $T = 1287$ lb-ft

$T = 1.29$ kip-ft