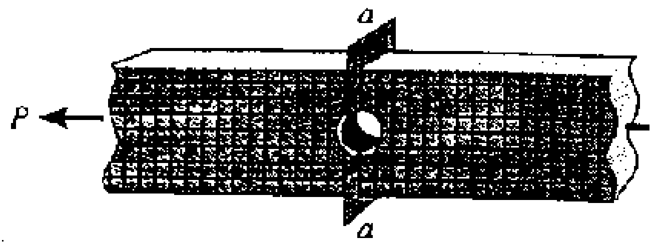
Chapter 3.

Generalized Hook's Law

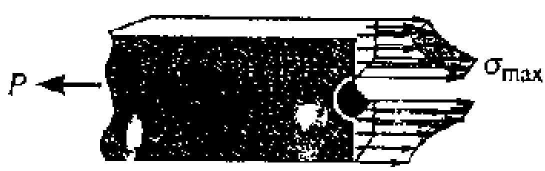
4.7 Stress Concentrations



Undistorted

Distorted

(a)

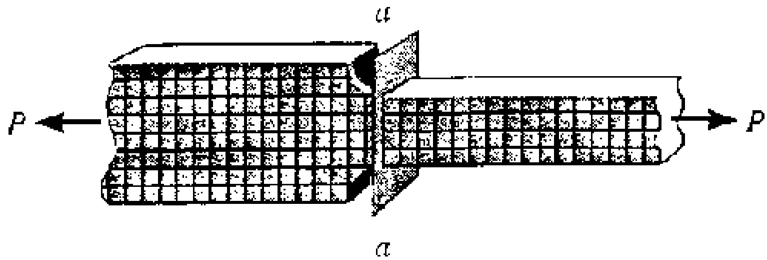


Actual stress distribution

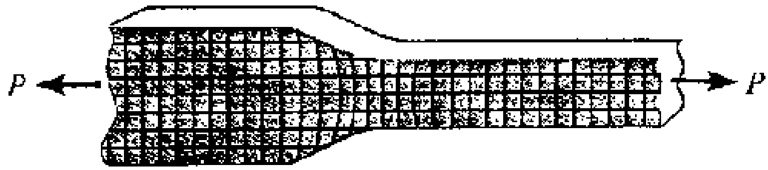
(b)

Average stress distribution

Prob. 4–21 (c)

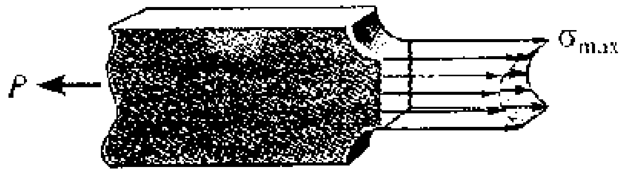


Undistorted



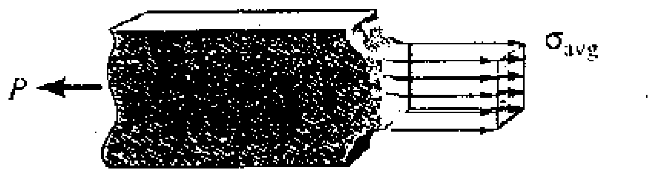
Distorted

(a)



Actual stress distribution

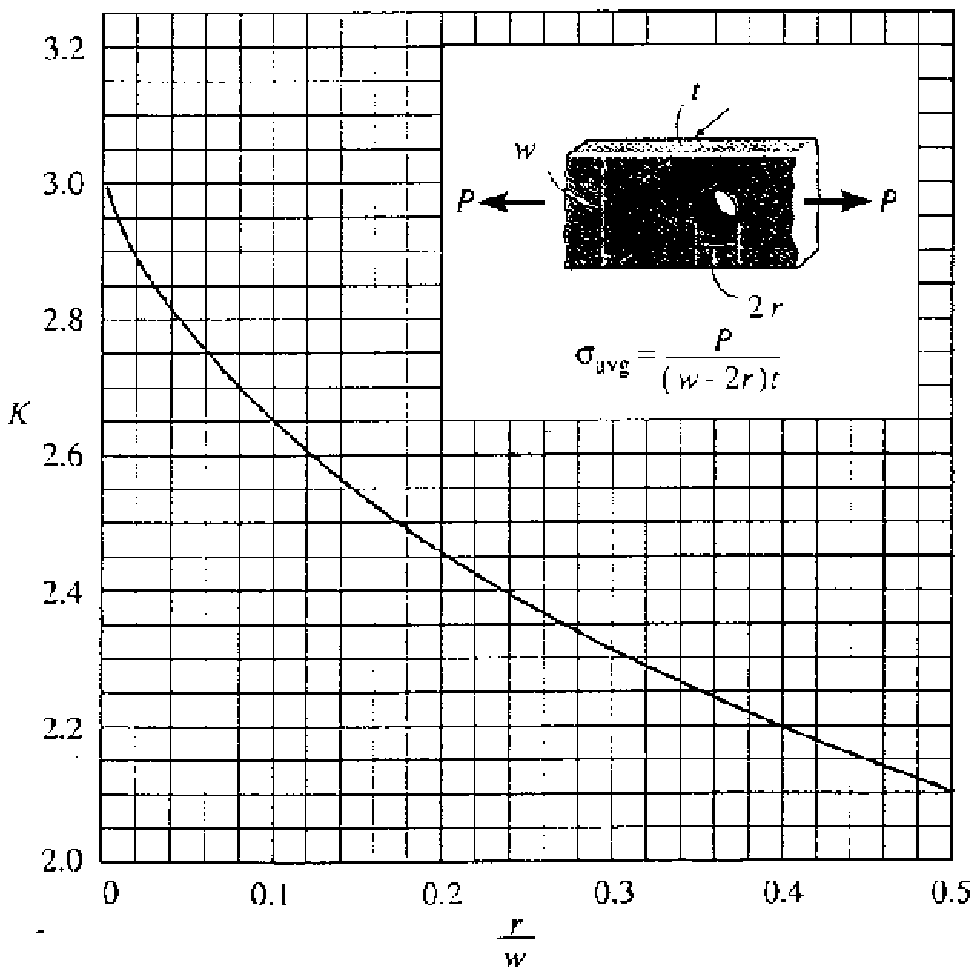
(b)



Average stress distribution

(c)

$$\boxed{K = \frac{\sigma_{max}}{\sigma_{avg}}} \qquad (4\text{–}7)$$



$$K = \frac{\sigma_{max}}{\sigma_{avg}}$$

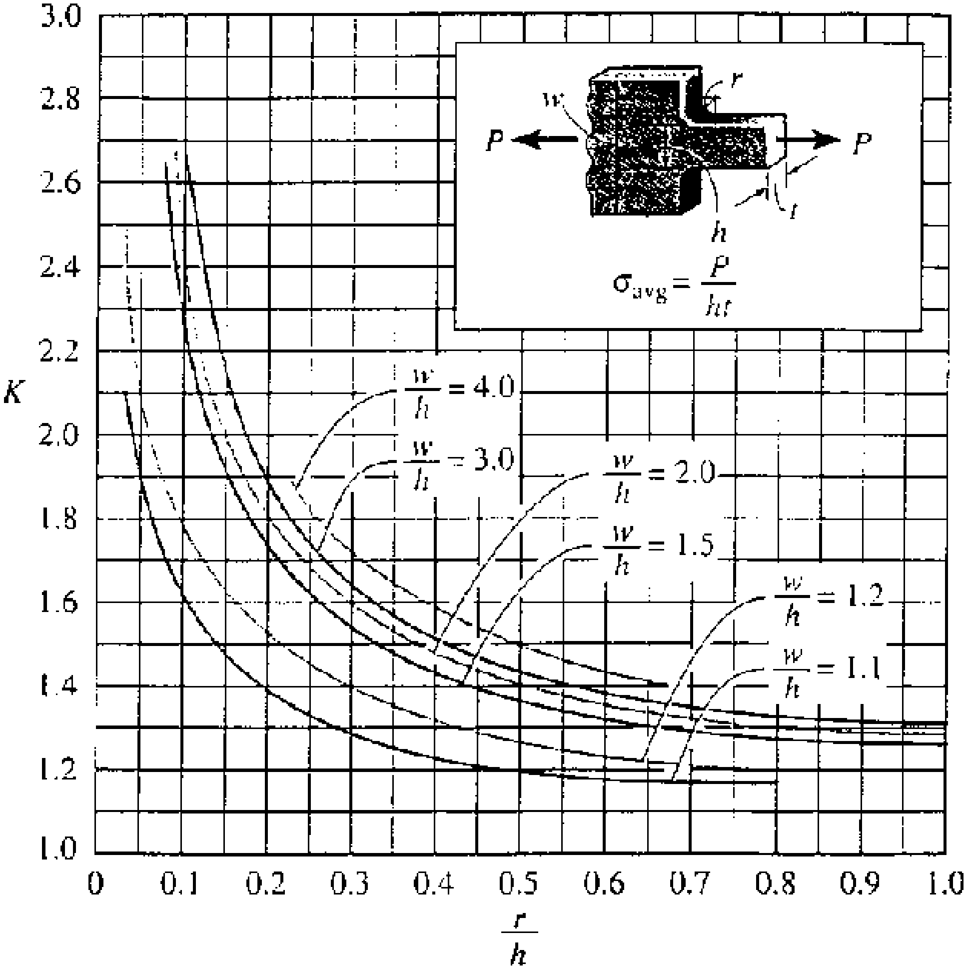
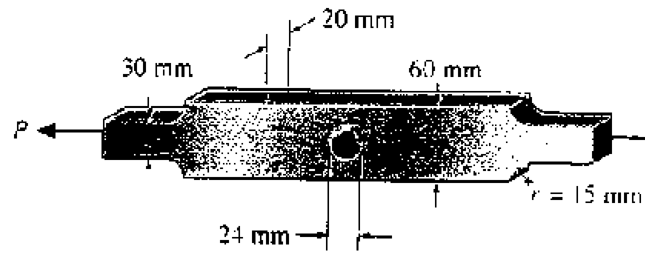
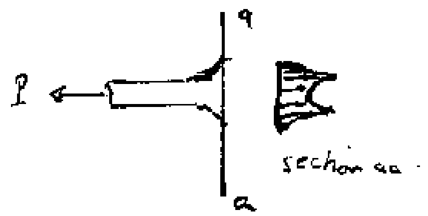


Fig. 4–23

4-95

4–95. The steel bar has the dimensions shown. Determine the maximum axial force P that can be applied so as not to exceed an allowable tensile stress of $\sigma_{allow} = 150$ MPa.





$W = 60$, $h = 30$ $\Big\}$ $\frac{W}{h} = 2$

$\frac{r}{h} = \frac{15}{30} = 0.5$

From chart $\left(\frac{W}{h} = 2,\ \frac{r}{h} = 0.5\right)$

$\sigma_{max} = \sigma_{all} = 150$

$\sigma_{max} = K\ \sigma_{avg}$

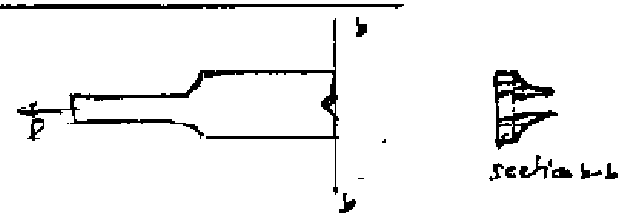
$150 = 1.4\ \sigma_{avg}$

$\sigma_{avg} = 107.14$

$P_1 = \sigma_{avg}\ A$

$P_1 = (107.14)(30)(20)$

$P_1 = 64.3\ kN$ ⇐



$\frac{2r}{W} = \frac{24}{60} = 0.4$

$K = 2.2$ (from chart)

$\sigma_{max} = K\ \sigma_{avg}$

$150 = 2.2\ \sigma_{avg}$

$\sigma_{avg} = 68.18$

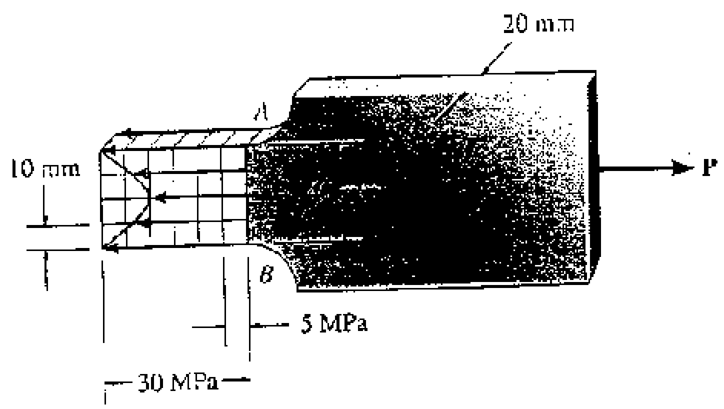
$\frac{P}{2} = \sigma_{avg}\ A_{net}$

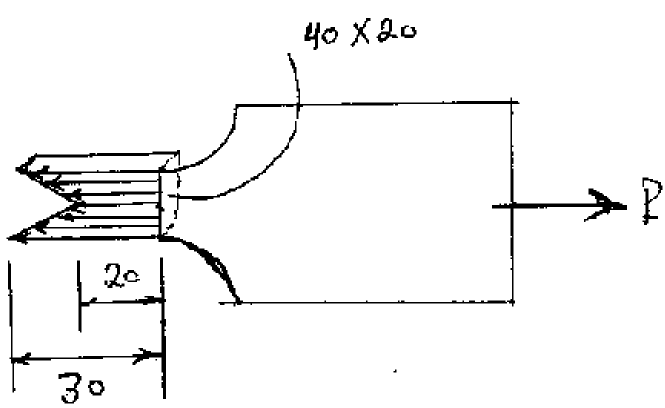
$= (68.18)(60-24)(20)$

$P_2 = 49\ kN$ ⇐

$P = P_2 = 49$ kN The smallest will control.

4–101. The resulting stress distribution along the section AB of the bar is shown in the figure. From this distribution, determine the approximate resultant axial force P applied to the bar. Also, what is the stress-concentration factor for this geometry?





$\sigma_{avg.} = (30 + 20)/2 = 25$

$\sigma_{max} = 30$

$K = \sigma_{max} / \sigma_{avg.} = 30/20 = 1.2$

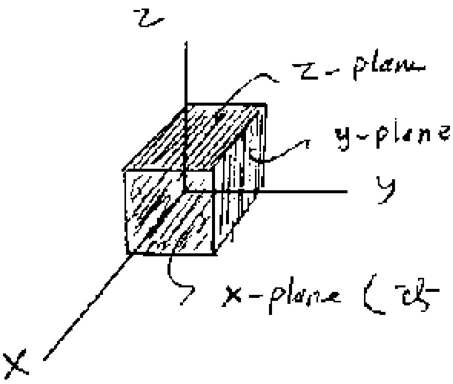
$P = \sigma_{avg.} A_{net}$

$= (25)(40)(20)$

$= 20 \text{ kN}$

General State of Stress

a plane is defined by its normal



σ_{ij} or τ_{ij} is the stress component which act on the ith plane in the jth direction

plane — direction

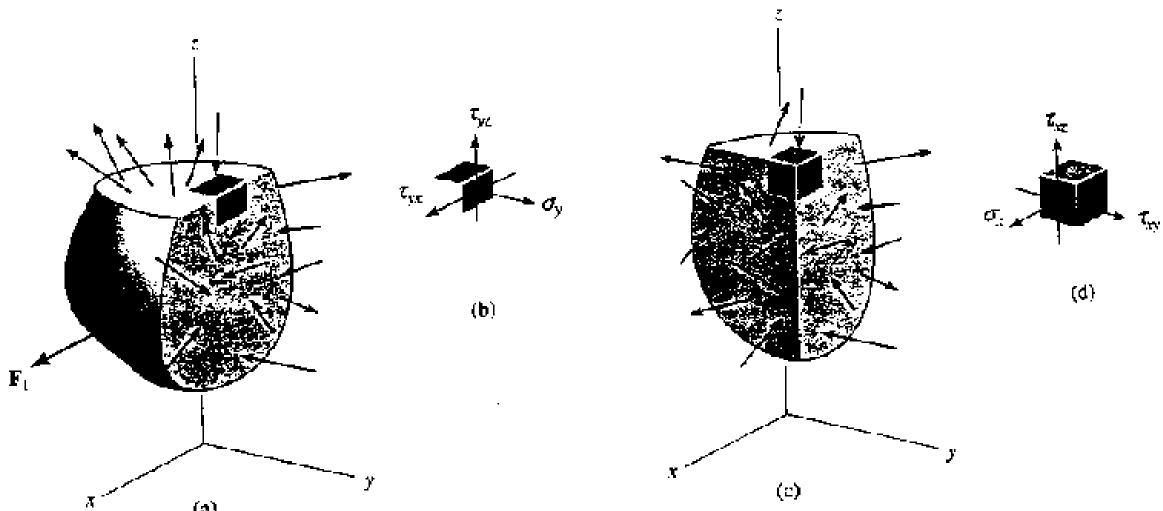
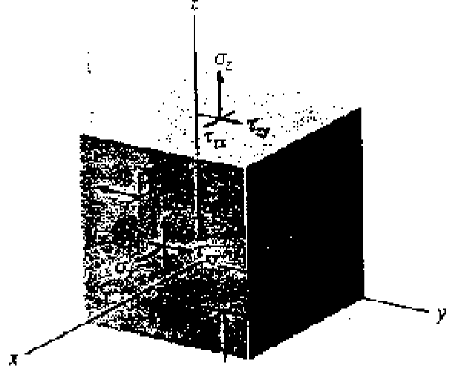


Fig. 1-11



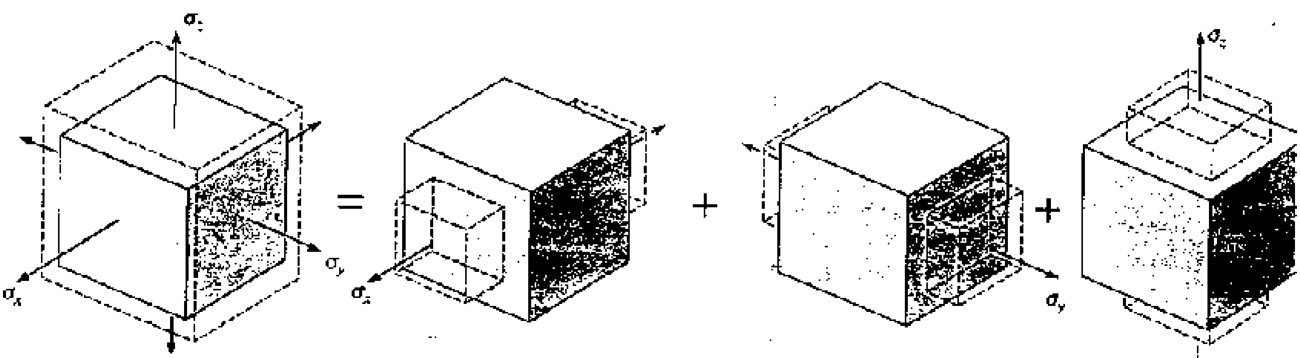
General state of stress

you see only positive planes

For equilibrium $\boxed{\tau_{ij} = \tau_{ji}}$

$\tau_{xy} = \tau_{yx}$

Generalized Hook's Law

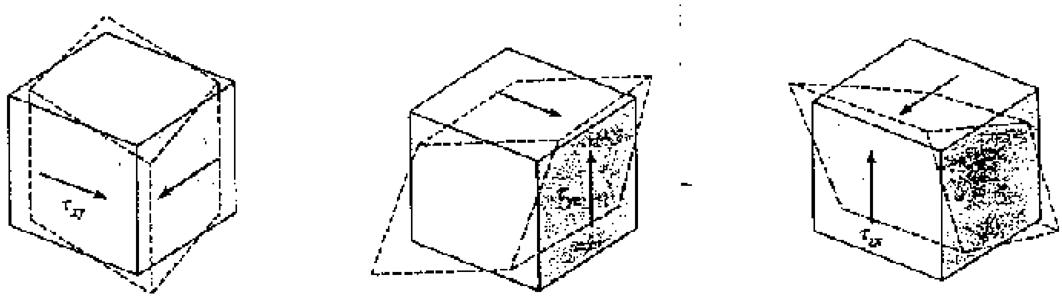


(a)

$$\epsilon'_x = \frac{\sigma_x}{E} \qquad \epsilon''_x = -\nu\frac{\sigma_y}{E} \qquad \epsilon'''_x = -\nu\frac{\sigma_z}{E}$$

$$\epsilon = \epsilon'_x + \epsilon''_x + \epsilon'''_x$$

$$\boxed{\begin{aligned} \epsilon_x &= \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] \\ \epsilon_y &= \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] \\ \epsilon_z &= \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] \end{aligned}}$$

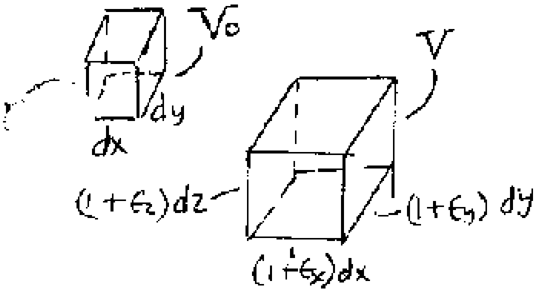


(a) (b) (c)

$$\boxed{\gamma_{xy} = \frac{1}{G}\tau_{xy} \qquad \gamma_{yz} = \frac{1}{G}\tau_{yz} \qquad \gamma_{zx} = \frac{1}{G}\tau_{zx}}$$

$$\boxed{G = \frac{E}{2(1+\nu)}}$$

Dilatation and Bulk Modulus



$V_o = dx\,dy\,dz$, original volume of element

$V = (1+\epsilon_x)(1+\epsilon_y)(1+\epsilon_z)\,dx\,dy\,dz$, volume of element after def

$\Delta V = (\epsilon_x + \epsilon_y + \epsilon_z)\,dx\,dy\,dz$

$$\boxed{e = \frac{\Delta V}{V_o} = \epsilon_x + \epsilon_y + \epsilon_z}$$

Volumetric strain, or dilatation

From Hook's law (add three equations)

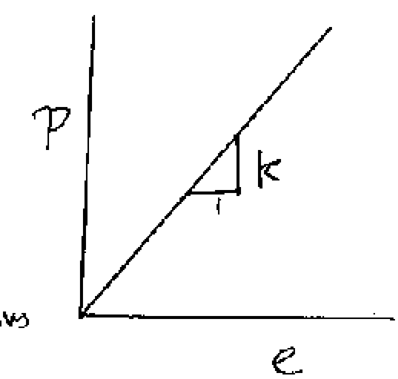
$$e = \frac{1-2\nu}{E}\,(\sigma_x + \sigma_y + \sigma_z)$$

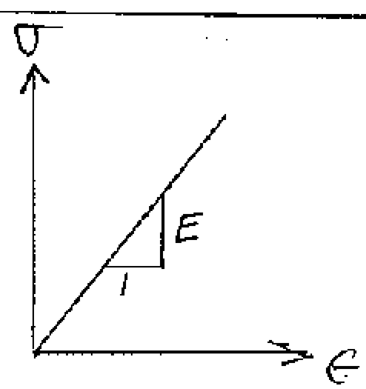
If hydrostatic pressure is calculated as average normal stress $p = (\sigma_x + \sigma_y + \sigma_z)/3$, or

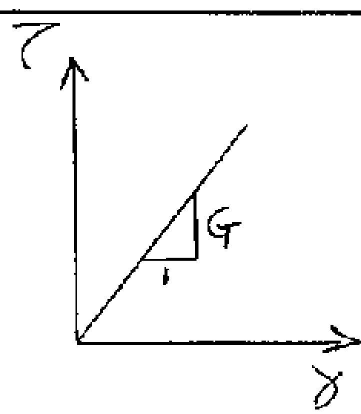
$$3p = (\sigma_x + \sigma_y + \sigma_z)$$

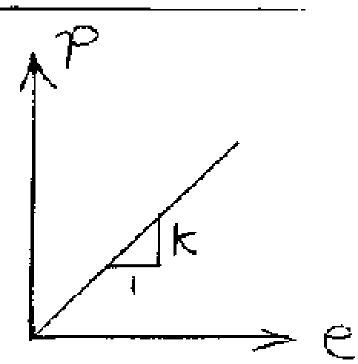
$$\therefore \quad e = \frac{1-2\nu}{E}\,3p \quad , \text{ or}$$

$$\frac{p}{e} = \boxed{\frac{E}{3(1-2\nu)} = K} \text{, Bulk modulus}$$









E — Modulus of elasticity
G — Modulus of rigidity
K — Bulk modulus
ν — Poisson's ratio

Given two, other two can be found

10-33 / 5

Given strains in terms stresses for (plane stress), 2-D.
Find stresses in terms of strains

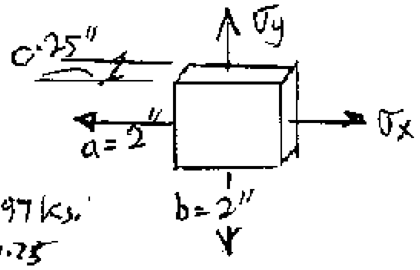
$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu\,\sigma_y}{E} \quad \text{——— (1)}$$
$$\epsilon_y = \frac{\sigma_y}{E} - \nu\,\frac{\sigma_x}{E} \quad \text{——— (2)}$$

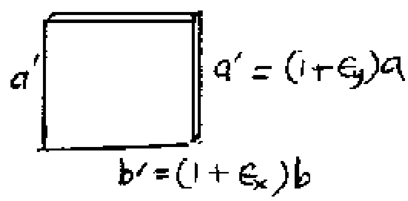
Multiply (1) by (ν) and add it (2) to eliminate σ_x and solve for σ_y $\Rightarrow$

$$\boxed{\sigma_y = \frac{E}{1-\nu^2}\left(\epsilon_y + \nu\,\epsilon_x\right)}$$

Multiply (2) by (ν) and add it to (1), to iliminate and solve for σ_x $\Rightarrow$ $\boxed{\sigma_x = \frac{E}{1-\nu^2}\left(\epsilon_x + \nu\,\epsilon_y\right)}$

10-40 / 520

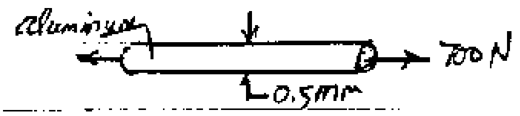




$E = 597$ ksi
$\nu = 0.75$

$$\sigma_x = \frac{500\,(2)}{2(0.25)} = 2\,\text{ksi}\ , \qquad \sigma_y = \frac{350\,(2)}{2(0.25)} = 1.4\,\text{ksi} \qquad \sigma_z = 0$$

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} = 2.76\times10^{-3} \Rightarrow b' = (1+2.76\times10^{-3})\,2 = 2.00553\ \text{in}$$
$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\nu\sigma_x}{E} = 1.50757\times10^{-3} \Rightarrow a' = (1+1.50757\times10^{-3})\,2 = 2.0302\ \text{in}$$
$$\epsilon_z = -\frac{\nu}{E}(\sigma_x+\sigma_y) = -1.42378\times10^{-3} \Rightarrow t' = (1+(-1.42378\times10^{-3}))(0.25) = 0.2496\ \text{in}$$



determine principal strains

10-4

$E = 73.1$ GPa
$P = 700$ N
$d = 20$ mm
$\nu = 0.35$

$$\sigma_x = P/A = 700/\left(\frac{\pi(20)^2}{4}\right) = 2.23\ \text{MPa}\ , \quad \sigma_y = \sigma_z = 0$$
$$\epsilon_x = \frac{\sigma_x}{E} = \frac{2.23}{73.1\times10^{3}} = 30.5\times10^{-6}$$
$$\epsilon_y = \epsilon_z = -\frac{\nu\sigma_x}{E} = (-0.35)(30.5\times10^{-6}) = -10.67\times10^{-6}$$