Chapter 2.

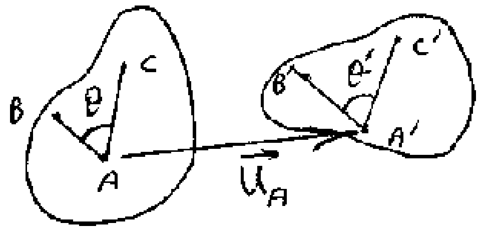
Deformation

2.1 Deformation

Deformation is the change in volume and shape of bodies when subjected to forces.

Deformation is noticable for materials like rubber whereas it is hard to be seen for structural members like reinforced concrete ... why ?

Displacement : is a vector quantity which measure the movement of a particle or a point in a body from one position to another.



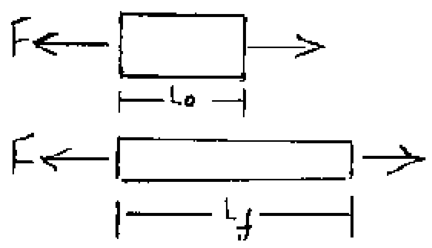
$A'B'$ may $\neq AB$
$A'C'$ may $\neq AC$
$B'C'$ may $\neq BC$
θ' may $\neq \theta$

$\vec{u}_A$ is the displacement vector of point A.

The best way to quantify deformation is to measure strains

2.2 Strains

As there are two kinds of stresses, there are two kinds of strain. Normal strains caused by Normal stresses and shear strains caused by shear stresses.

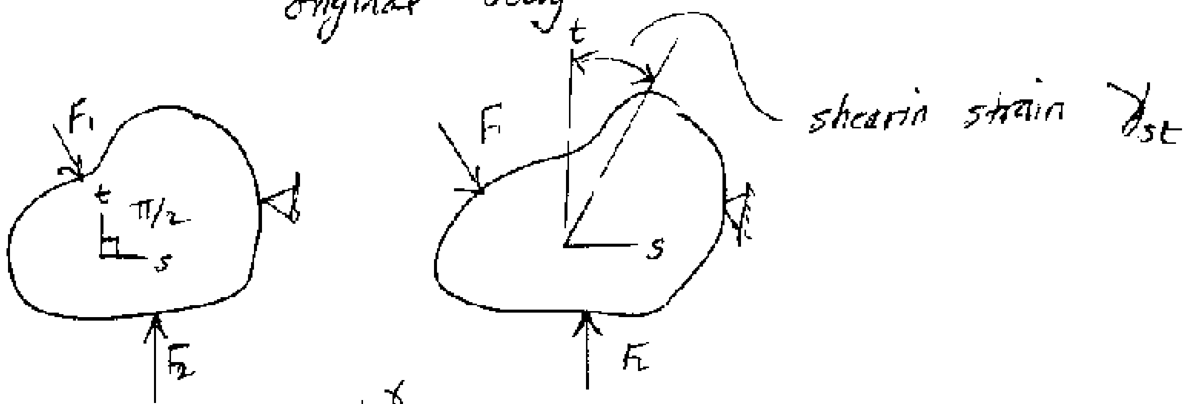


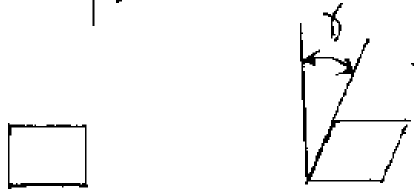
L_o = original length
L_f = final length
$\Delta L = \delta$ = change in length

$$\text{Normal strain} = \frac{\text{change in length}}{\text{original length}} \quad \left(\epsilon = \frac{\Delta L}{L_o} = \frac{\delta}{L_o} \right)$$

$$\boxed{L_f = (1+\epsilon)\, L_o}$$

<u>Shear strain</u> : the chang in the 90° angle in the original body.





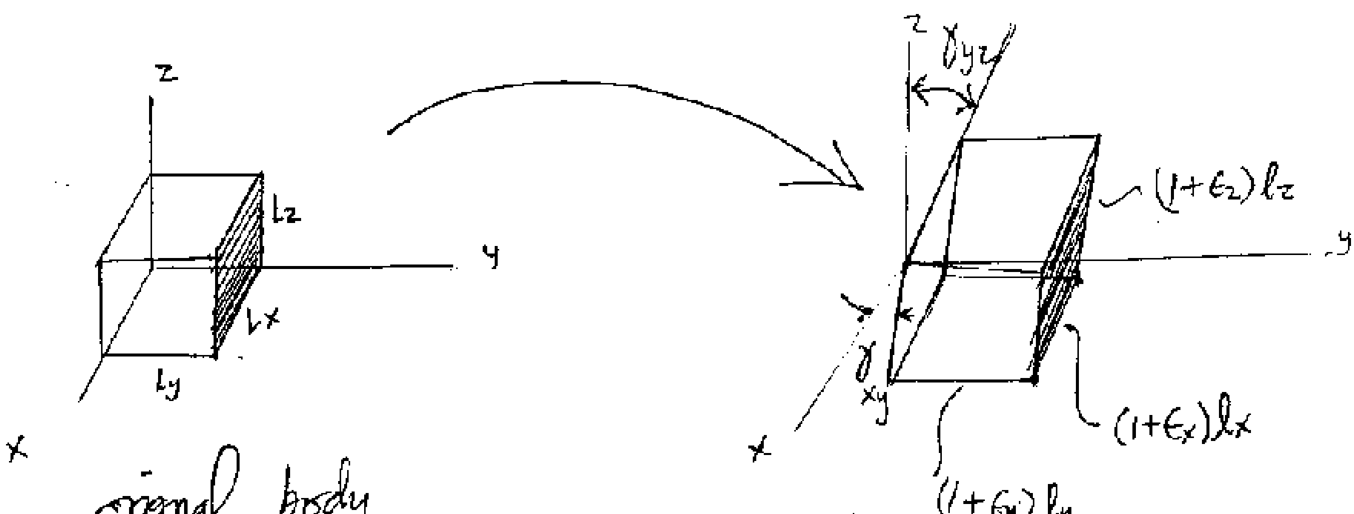
as $\gamma \ll 1$

$\therefore \quad \sin\gamma = \gamma$

$\cos\gamma = 1$

& $\tan\gamma = \dfrac{\sin\gamma}{\cos\gamma} = \gamma$

Change of shape and volume



orignal body

deformed body.

3.1 The Tension and Compression Test

One of the most important test to perform is the tension or compression test which is meant to establish the relationship between normal stress σ and normal strain ϵ using a standard specimen.

① Gage length L_0 : is the length along which strain is monitored.

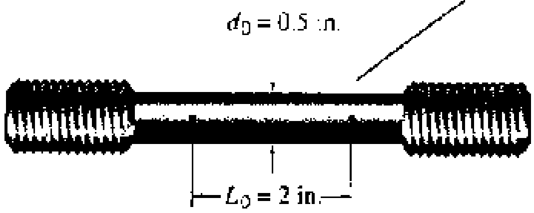


Fig. 3–1

② Extensometer : is a device to measure change in length

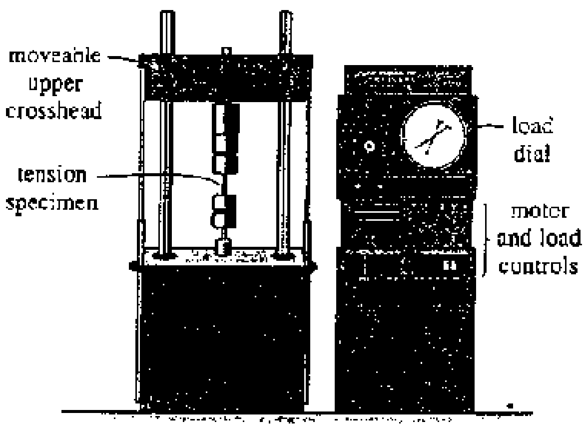


Fig. 3–2

strain gage : an electrical device to measure strain (normal strain), directly

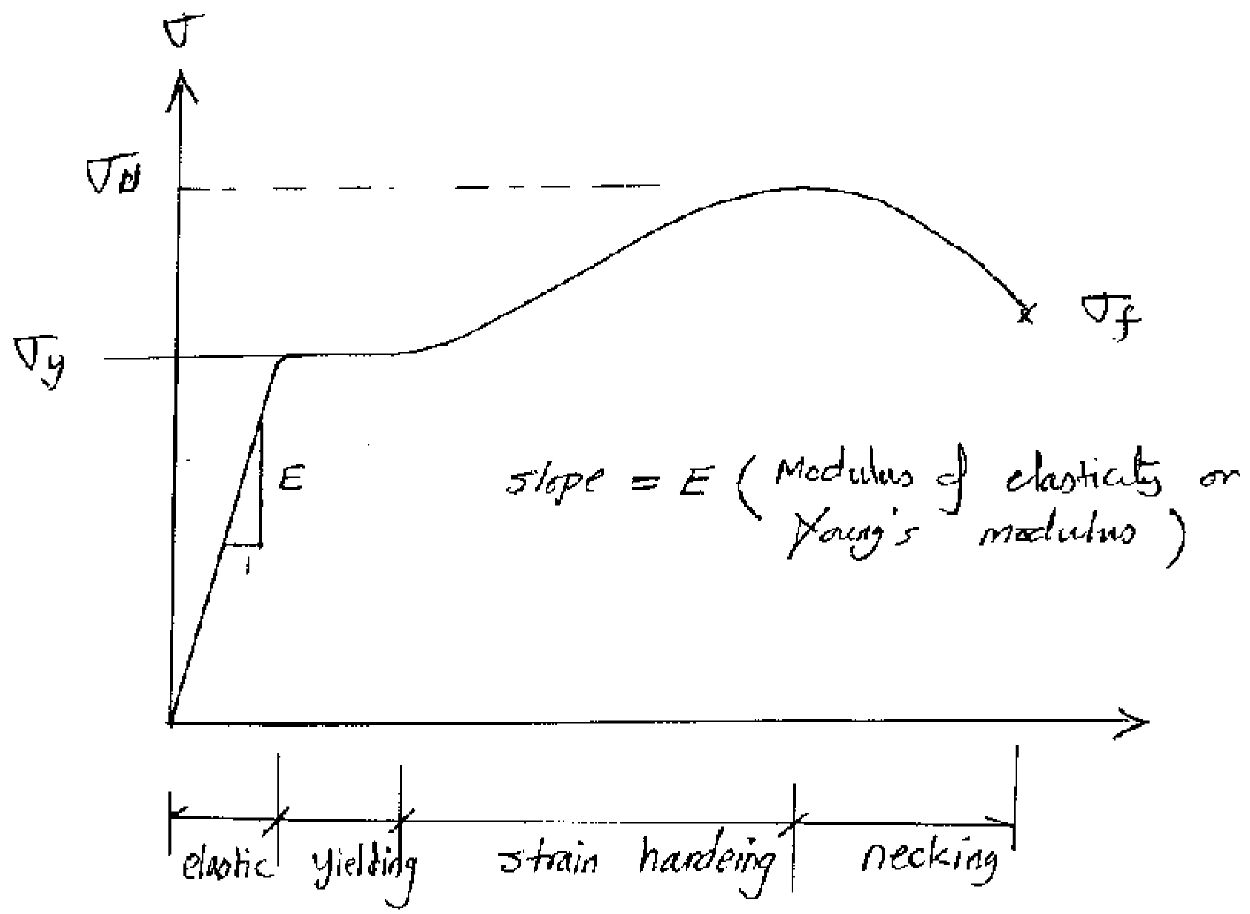
The end product of the test is a stress-strain curve from which material properties can be calculated.

3-2 The Stress - Strain Diagram.

Using a standard specimen of known gage length L_0 and of original cross section area A_0 and run a tension test according to ASTM (American Society for Testing and Materials) speifications. This required applying a force P with a specified rate then.

- Calculate normal stress $\sigma = \frac{P}{A_0}$
- Calculate normal strain $\epsilon = \Delta L / L_0$
- plot stress versus strain,

one may have a curve like this



stress - strain curve for ductile steel.

3.3 Stress - Strain Behavior of Ductile and Brittle materials

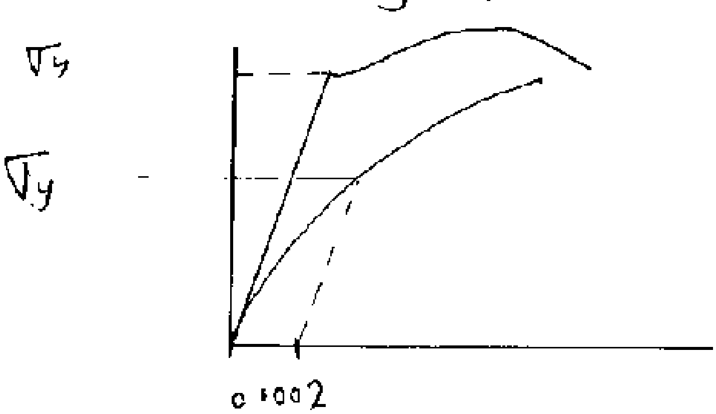
Ductile materials : are those materials which exhibit large deformation before failing.

Ductility is measured by one of two :

$$\% \text{ elongation} = \frac{L_f - L_o}{L_o} \times 100$$

$$\% \text{ reduction in area} = \frac{A_o - A_f}{A_o} \times 100$$

For materials which does not posses definite yield point, the yield stress (strength) is obtained by the use of offset method which is normally at 0.002 strain



Brittle materials : are those which exhibit little or no yielding before failing

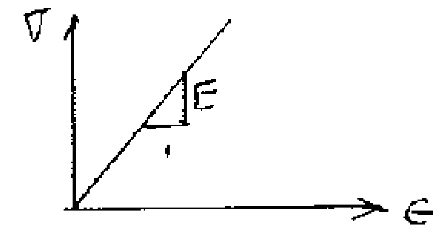
examples { Concrete and cast iron }

See fig. 3.9 and fig 3.11 in your textbook pages (92 & 93).

3.4 Hooke's Law.

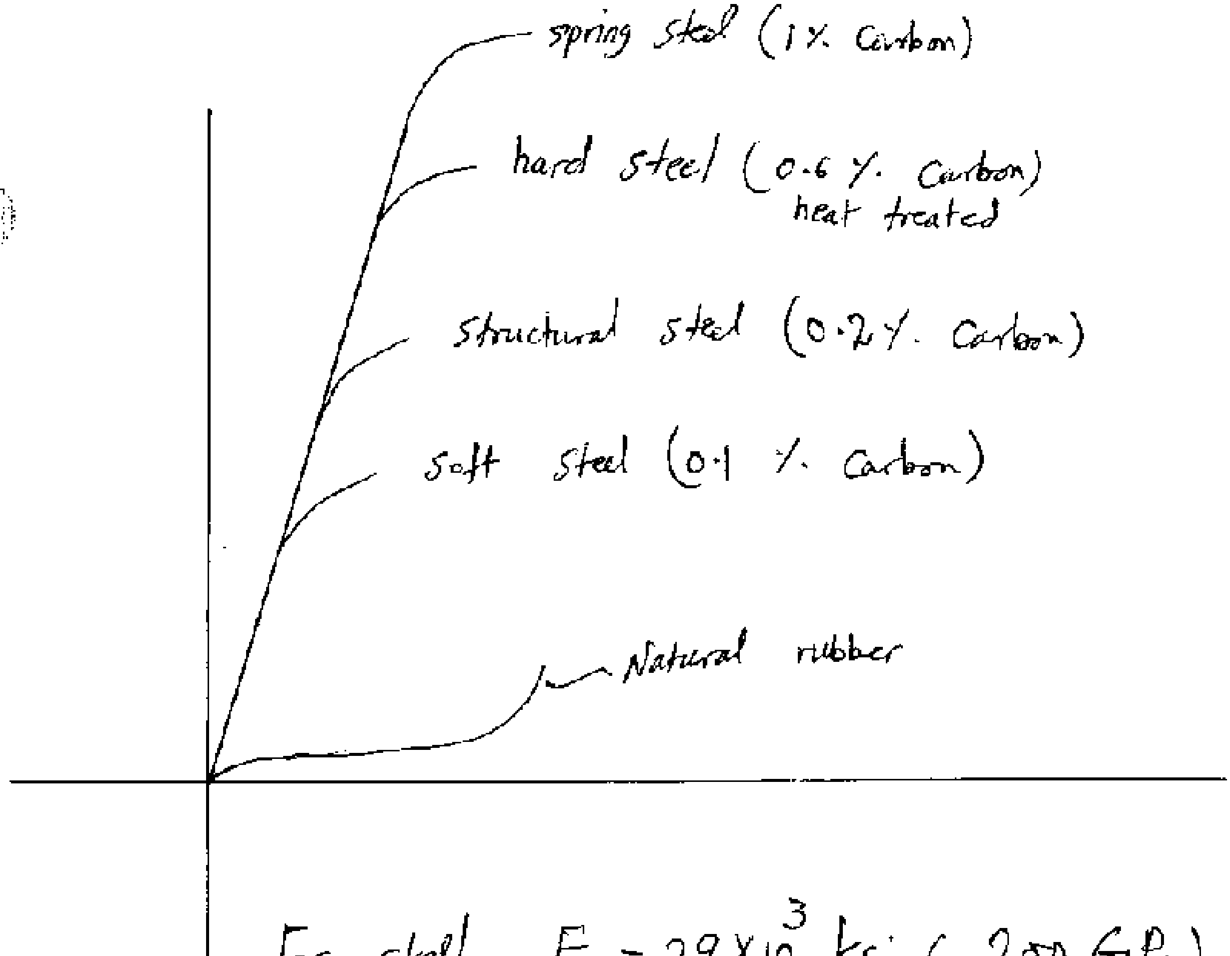
The linear stress-strain relationship is called Hooke's law.

$$\boxed{\sigma = E\epsilon}$$



Remember E is called modulus of elasticity.

Remark: Although one may vary the yield strength of steel by varying the % of carbon, the modulus of elasticity remain the same as in steel.

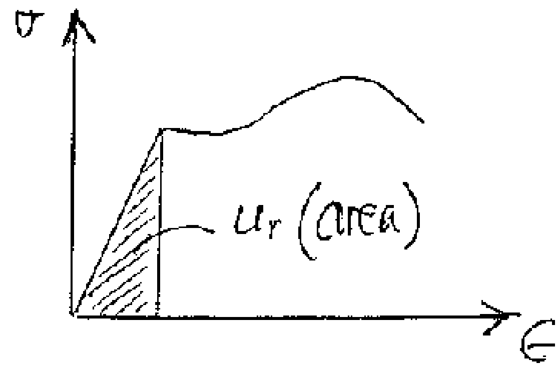


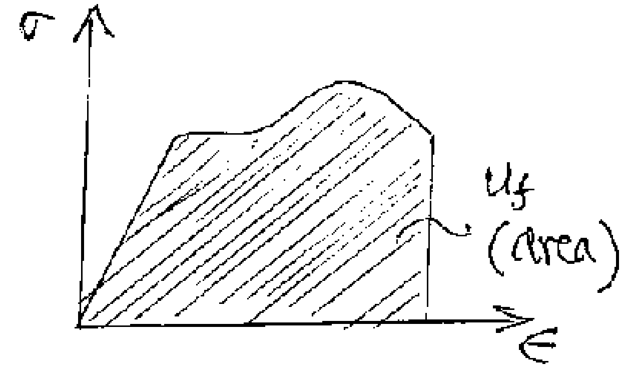
For steel $E = 29 \times 10^3$ ksi (200 GPa)

For rubber $E = 0.1 \times 10^3$ ksi (0.7 MPa)

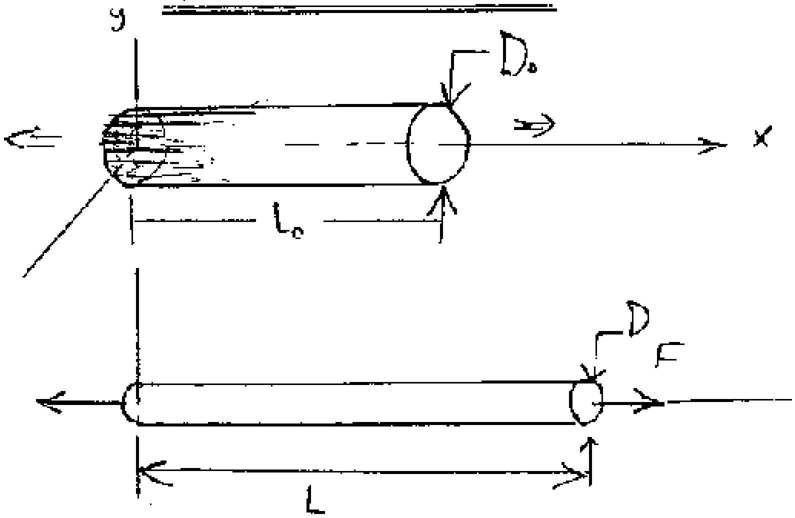
That is why deformation in rubber is noticable.

There are two more important properties for materials one is called Modulus of resilience U_r and the other is modulus of toughness U_f





3.6 Poisson's Ratio ν

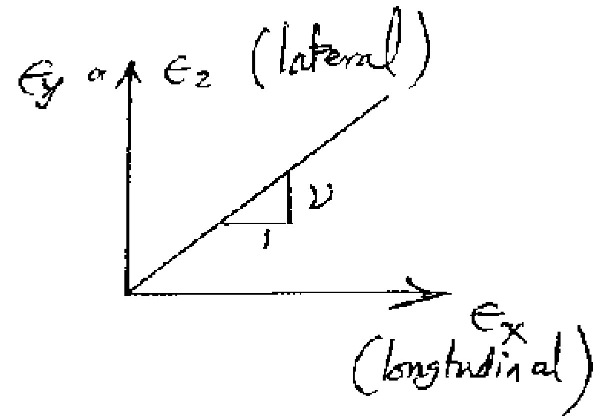


L_o = original length
D_o = original diameter
D = current diameter
L = current length.

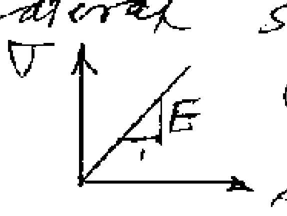
$$\epsilon_x = \frac{L - L_o}{L} > 0$$

$$\epsilon_y = \epsilon_z = \frac{D - D_o}{D_o} < 0$$

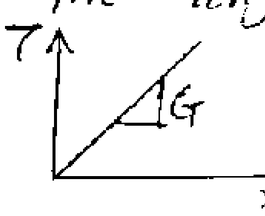
$$\epsilon_y \text{ or } \epsilon_z = -\nu\,\epsilon_x$$



Poisson's ratio ν is nothing but the relation between lateral strain to the longitudinal strain



$$\sigma = E\epsilon$$



$$\tau = G\gamma$$

Modulus of rigidity

Average Mechanical Properties of Typical Engineering Materials[a]
(U.S. Customary Units)

Materials		Specific Weight γ (lb/in³)	Modulus of Elasticity E (10³) ksi	Modulus of Rigidity G (10³) ksi	Yield Strength (ksi) σ_y Tens.	Comp.[b]	Shear	Ultimate Strength (ksi) σ_u Tens.	Comp.[b]	Shear	% Elongation in 2 in. specimen	Poisson's Ratio ν	Coef. of Therm. Expansion α $(10^{-6})/°F$
Metallic													
Aluminum Wrought Alloys	2014-T6	0.101	10.6	3.9	60	60	25	68	68	42	10	0.35	12.8
	6061-T6	0.098	10.0	3.7	37	37	19	42	42	27	12	0.35	13.1
Cast Iron Alloys	Gray ASTM 20	0.260	10.0	3.9	—	—	—	26	97	—	0.6	0.28	6.70
	Malleable ASTM A-197	0.263	25.0	9.8		—	—	40	83	—	5	0.28	6.60
Copper Alloys	Red Brass C83400	0.316	14.6	5.4	11.4	11.4	—	35	35	—	35	0.35	9.80
	Bronze C86100	0.319	15.0	5.6	50	50		95	95	—	20	0.34	9.60
Magnesium Alloy	[Am 1004-T61]	0.066	6.48	2.5	22	22	—	40	40	22	1	0.30	14.3
Steel Alloys	Structural A36	0.284	29.0	11.0	36	36	—	58	58		30	0.32	6.60
	Stainless 304	0.284	28.0	11.0	30	30	—	75	75	—	40	0.27	9.60
	Tool L2	0.295	29.0	11.0	102	102	—	116	116	—	22	0.32	6.50
Titanium Alloy	[Ti 6Al 4V]	0.160	17.4	6.4	134	134	—	145	145	—	16	0.36	5.20
Nonmetallic													
Concrete	Low Strength	0.086	3.20	—	—	—	1.8	—	—	—		0.15	6.0
	High Strength	0.086	4.20		—	—	5.5	—	—	—	—	0.15	6.0
Plastic Reinforced	Kevlar 49	0.0524	19.0		—	—	—	104	70	10.2	2.8	0.34	
	30% Glass	0.0524	10.5	—	—	—		13	19	—	—	0.34	—
Wood Select Structural Grade	Douglas Fir	0.017	1.90	—	—	—		0.30[c]	3.78[d]	0.90[d]	—	0.29[e]	—
	White Spruce	0.130	1.40	—	—	—	—	0.36[c]	5.18[d]	0.97[d]	—	0.31[e]	—

[a] Specific values may vary for a particular material due to alloy or mineral composition, mechanical working of the specimen, or heat treatment. For a more exact value reference books for the material should be consulted.

[b] The yield and ultimate strengths for ductile materials can be assumed equal for both tension and compression.

[c] Measured perpendicular to the grain.

[d] Measured parallel to the grain.

[e] Deformation measured perpendicular to the grain when the load is applied along the grain.

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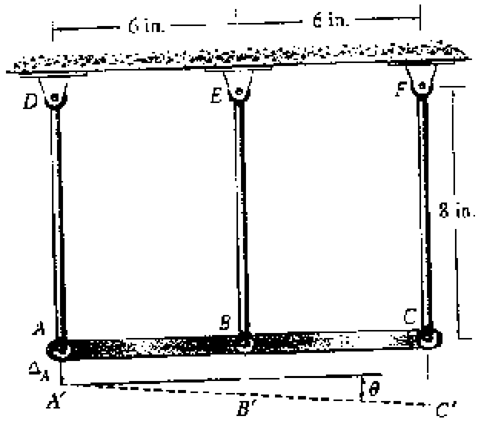
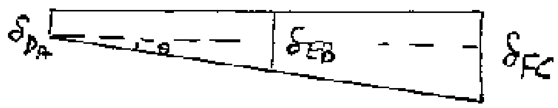
[c] Measured perpendicular to the grain.

[d] Measured parallel to the grain.

[e] Deformation measured perpendicular to the grain when the load is applied along the grain.

2–3. Bar ABC is originally in a horizontal position. If loads cause the end A to be displaced downwards $\Delta_A = 0.002$ in. and the bar rotates $\theta = 0.2°$, determine the average normal strain in the rods AD, BE, and CF.





$\delta_{DA} = 0.002$ in

$$\tan\theta = \frac{\delta_{EB} - 0.002}{6} = \tan 0.2° \implies \delta_{EB} = 0.02294 \text{ in}$$

$$\tan\theta = \frac{\delta_{FC} - 0.002}{12} = \tan 0.2° \implies \delta_{FC} = 0.04388 \text{ in}$$

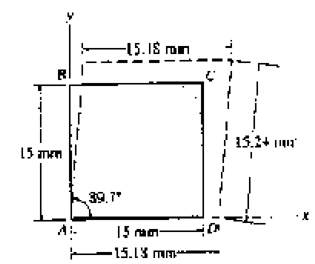
$$\epsilon_{AD} = \frac{\delta_{AD}}{l_{AD}} = \frac{0.002}{8} = 2.5 \times 10^{-4} \text{ in/in}$$

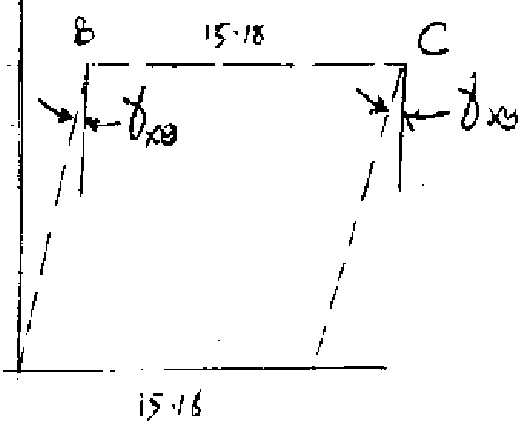
$$\epsilon_{BE} = \frac{\delta_{BE}}{l_{BE}} = \frac{0.02294}{8} = 2.8675 \times 10^{-3} \text{ in/in}$$

$$\epsilon_{FC} = \frac{\delta_{FC}}{l_{CF}} = \frac{0.04388}{8} = 5.485 \times 10^{-3} \text{ in/in}$$

2-22
79

2–22. A square piece of material is deformed into the dashed position. Determine the shear strain γ_{xy} at corners B and C.





$$\gamma_{xy} = -\frac{0.3°\pi}{180} \quad \text{at } B$$

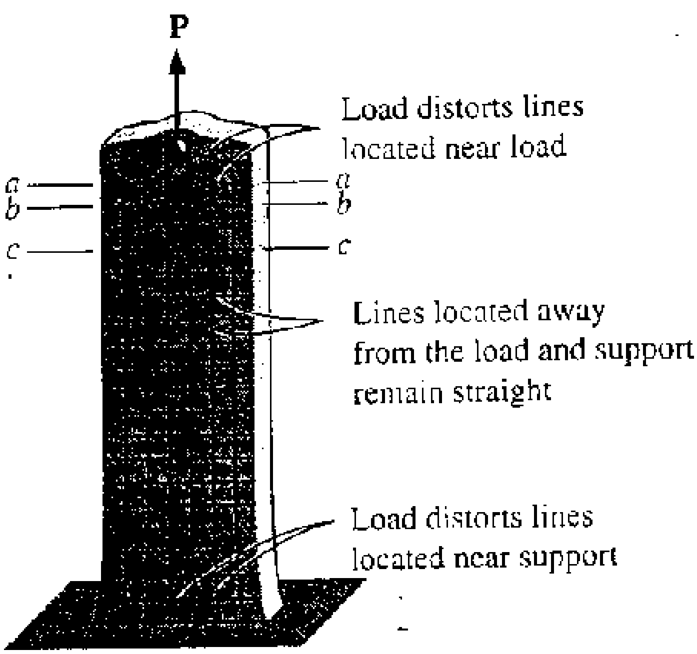
$$\gamma_{xy} = +\frac{0.3°\pi}{180} \quad \text{at } C$$

both of which $= 5.24 \times 10^{-3}$ rad.

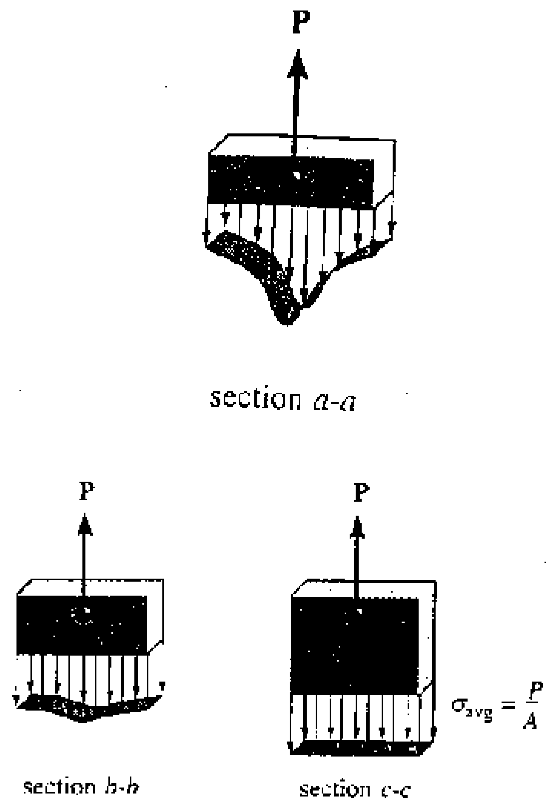
$\gamma = \tan 0.3° \approx 5.24 \times 10^{-3}$ rad

Ch. 4 Axial Load

4.1 <u>Saint - Venant Principal</u> : The stress tends to be large and non-uniform near or at support and where the load is applied and the stress tends to flatten and becomes uniform at a distance equal to the largest dimension away from support and load application.

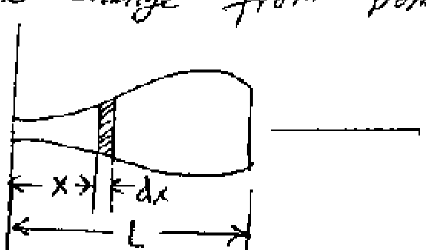


(a)



(b)

Just like stress, strain is define at a point which may varies and change from point to point.



$d\delta$ = elongation of element dx

δ = elongation of the whole member

$\therefore$ strain of element dx $\quad \epsilon(x) = \frac{d\delta}{dx}$ $\quad$ and $\quad \boxed{\delta = \int_0^L \epsilon(x)\, dx}$

for constant stress and strain $\quad \epsilon(x) = \epsilon = \text{constant}$ and the above becomes $\quad \underline{\delta = \epsilon L}$

for a member made of material whose Modulus of elasticity E:

$$\sigma(x) = E\epsilon(x) \quad \text{— Hook's law}$$

$$\frac{N(x)}{A(x)} = E\,\epsilon(x) \quad \text{— for variable load and area}$$

$\therefore$ $\epsilon(x) = \dfrac{N(x)}{E A(x)}$, which upon substitution, one has:

$$\delta = \int_0^L \frac{N(x)}{E\,A(x)}\,dx$$

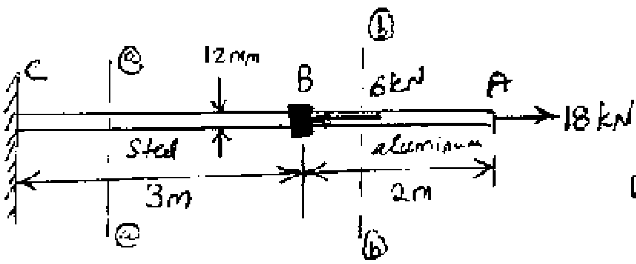
For constant $N(x)$ and $A(x)$: $N(x) = N$, $A(x) = A$ $\Rightarrow$

$$\delta = \frac{NL}{EA}$$

, for member composed of different materials or different segments of different area, then

$$\delta = \sum_{i=1}^{n} \frac{N_i L_i}{E_i A_i}$$ where n is number of segments.

Prob. 4-1
30



$E_{st} = 200$ GPa

$E_{al} = 70$ GPa

Determine displacement of B and the end A.

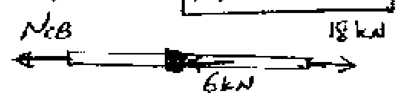
$$\delta_B = \delta_{BC} = \frac{N_{CB}\,l_{CB}}{E_{st}\,A_{CB}} = \frac{(12\times10^3)(3)}{(200\times10^9)\left(\frac{\pi(.012)^2}{4}\right)}$$

$$\delta_B = 0.00159 \text{ m}$$
$$= 1.59 \text{ mm}$$

section b-b

$\Sigma F_x = 0$ $\boxed{N_{AB} = 18\times10^3 \text{ N}}$

section a-a



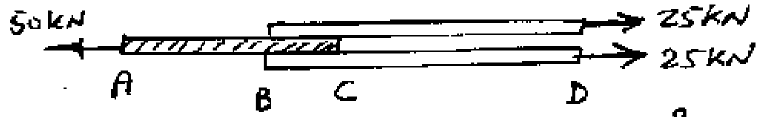
$\Sigma F_x = 18 - 6 - N_{CB} = 0$

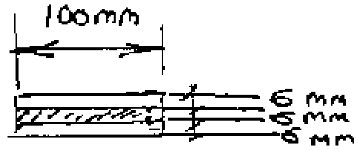
$\boxed{N_{CB} = 12\times10^3 \text{ N}}$

$$\delta_A = \delta_{CB} + \delta_{BA}$$
$$= 0.00159 + \frac{N_{AB}\,l_{AB}}{E_{al}\,A_{AB}} = 0.0015 + \frac{(18\times10^3)(2)}{(70\times10^9)\left(\frac{\pi(.012)^2}{4}\right)}$$

$$= 0.00614 \text{ m}$$
$$= 6.14 \text{ mm}$$

4-8 / 128





A-36 steel $\Rightarrow$ $E = 200 \times 10^9$

$L_{AB} = 600$ mm

$L_{BC} = 200$ mm

$L_{CD} = 800$ mm

$\delta_{A/D} = ?$

$$\delta = \frac{NL}{EA}$$

$$\delta_{A/D} = \delta_{A/B} + \delta_{B/C} + \delta_{C/D}$$

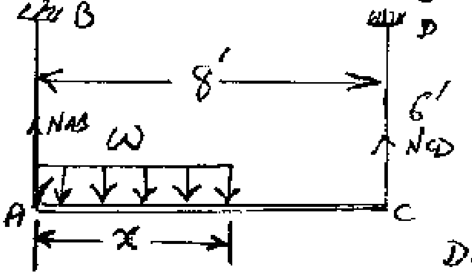
$$= \frac{N_{AB} L_{AB}}{E A_{AB}} + \frac{N_{BC} l_{BC}}{E A_{BC}} + \frac{N_{CD} l_{CD}}{E A_{CD}}$$

$$= \left\{ \frac{(50\times10^3)(0.6)}{(0.006)(0.1)} + \frac{(50\times10^3)(0.2)}{3(0.006)(0.1)} + \frac{(50\times10^3)(0.8)}{2(0.006)(0.1)} \right\} \frac{1}{29\times10}$$

$$= \frac{(50\times10^3)\{0.6 + 0.2/3 + 0.8/2\}}{(0.006)(0.1)\ 200\times10^9} = 4.44\times10^{-4}\ m$$

$$= \boxed{0.444\ mm}$$

4-21 / 130



AB & CD are steel wires

$d_{AB} = 0.5$ in , $d_{CD} = 0.3$ in

$\sigma_{all} = 16.2$ ksi

Determine ω & x such that AC remains horizontal.

$\Sigma F_y = 0:$ $\quad N_{AB} + N_{CD} = \omega x$ ——— (1)

$\Sigma M_{@A} = 0$ $\quad 8 N_{CD} = \frac{\omega x^2}{2}$ ——— (2)

AC remains horizontal : $\quad \delta_{AB} = \delta_{CD}$ $\quad \frac{N_{AB}(8)}{\frac{\pi(0.5)^2}{4}} = \frac{N_{CD}(6)}{\frac{\pi(0.3)^2}{4}}$ — (3)

$\sigma_{CD} = \sigma_{all}$: $\quad \frac{N_{CD}}{\frac{\pi(0.3)^2}{4}} = 16.2\times10^3$ ——— (4)

From (4) : $N_{CD} = 1145$ lb

From (3) : $N_{AB} = 3172$ lb

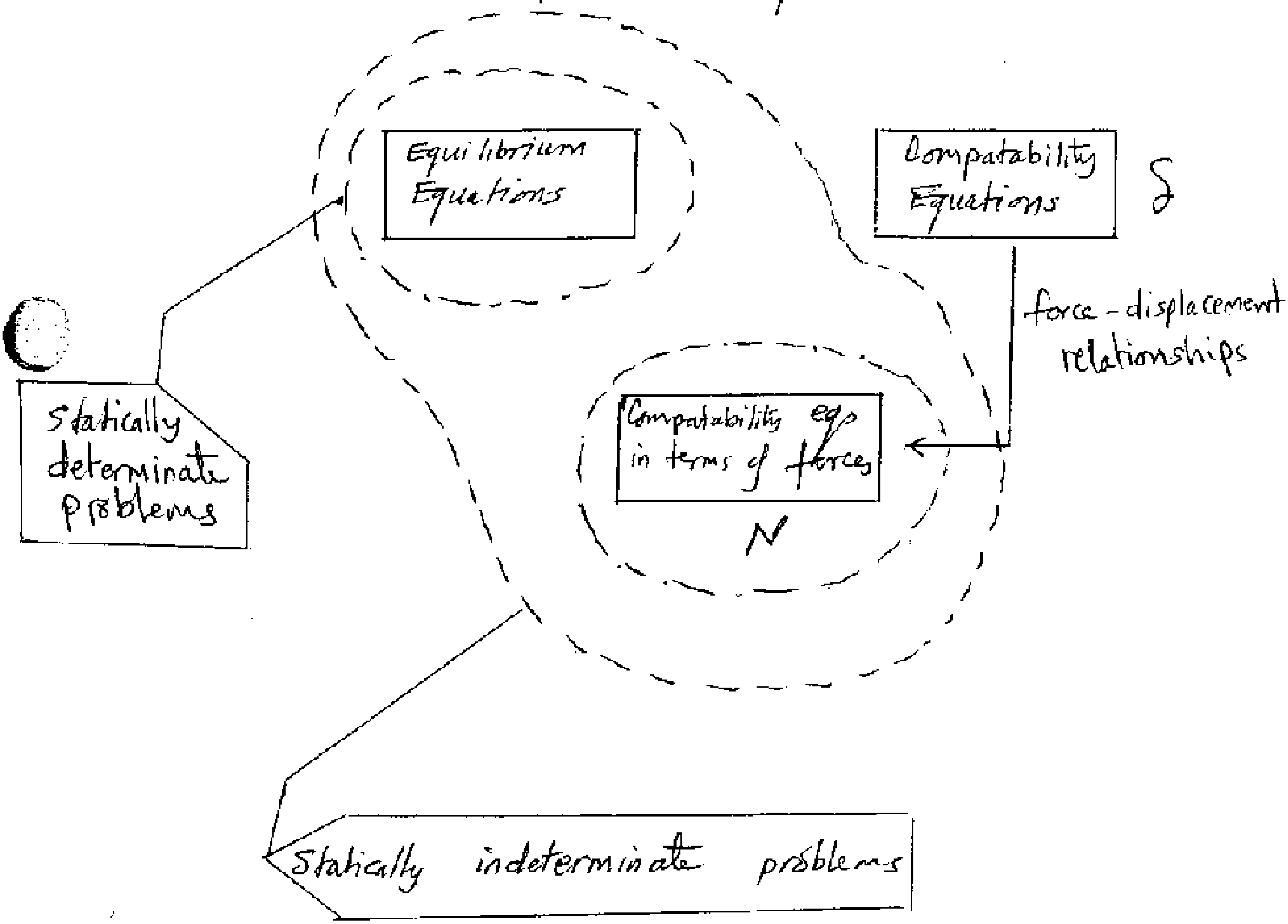
From (1) & (2) : $16 N_{CD} = (N_{AB} + N_{CD}) x$ $\Rightarrow$ $\underline{x = 4.24\ ft}$

from (1) : $\underline{\omega = 1.02\ kip/ft}$

Check that $\sigma_{AB} \le \sigma_{all}$: $\frac{N_{AB}}{\frac{\pi(0.5)^2}{4}} \le 16.2\times10^3$ $\quad$ ok

Statically Indeterminate of Axially loaded Members

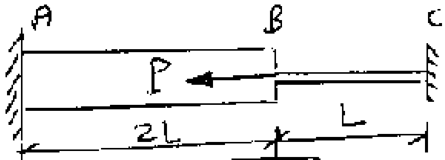
When number of unknowns exceeded number of equations of equilibrium, we say that the structure is statically indeterminate and thus the unknowns (reactions or internal normal forces) can not be obtained from equations of equilibrium alone and the one needs additional equations. These equations can be obtained from the deformation of the structure. These extra equations are known as compatability equations



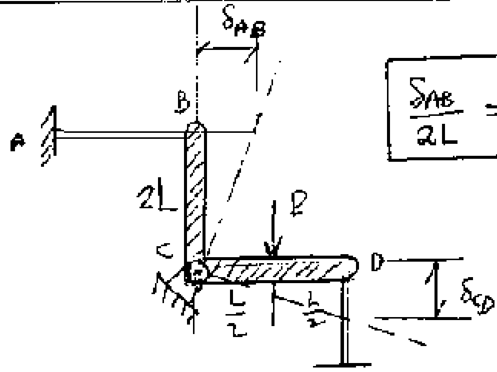
- Compatability equations are nothing but relation-ships between displacement of members which will be more useful if converted in terms of internal unknown forces which are used with eq. eqs to solve for unknowns.

Examples of Statically Indeterminate Problems

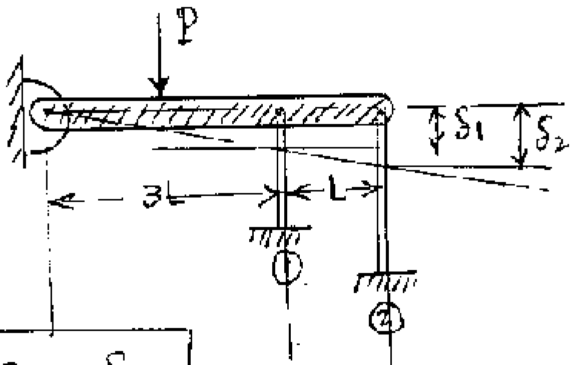
(P and L are given)



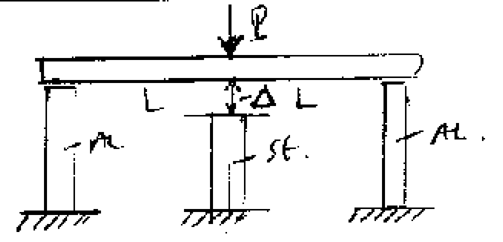
$$\boxed{\delta_{AB} + \delta_{BC} = 0}$$



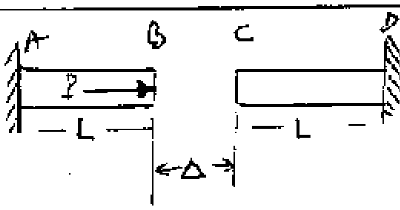
$$\boxed{\frac{\delta_{AB}}{2L} = \frac{\delta_{CD}}{L}}$$



$$\boxed{\frac{\delta_1}{3L} = \frac{\delta_2}{4L}}$$

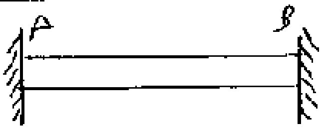


$$\boxed{\delta_{Al} = \delta_{st} + \Delta}$$

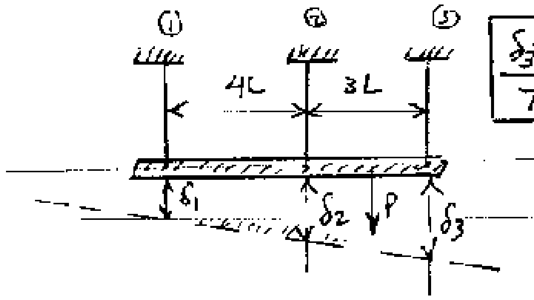


$$\boxed{\delta_{AB} + \delta_{CD} = \Delta}$$

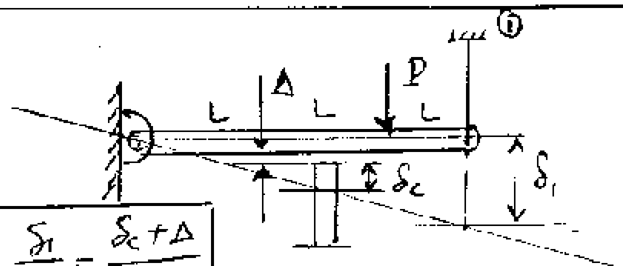
$$\boxed{\delta_{AB} = 0}$$



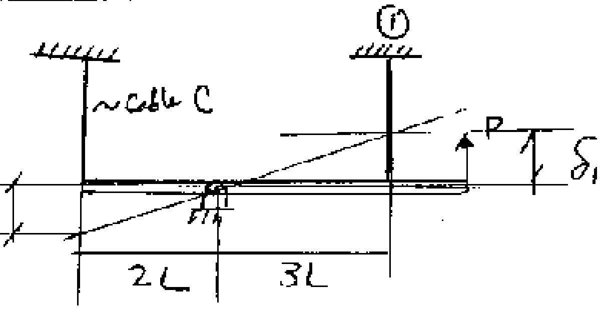
$\Delta T > 0$



$$\boxed{\frac{\delta_3 - \delta_1}{7L} = \frac{\delta_2 - \delta_1}{4L}}$$

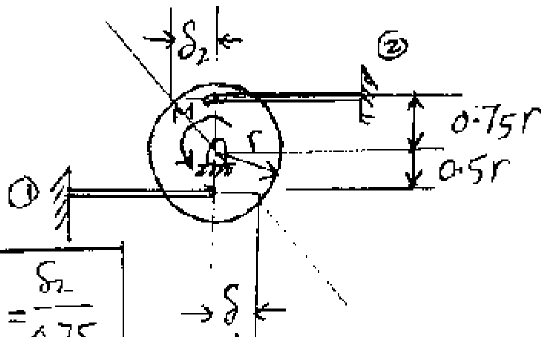


$$\boxed{\frac{\delta_1}{3L} = \frac{\delta_c + \Delta}{2L}}$$



$$\boxed{\frac{\delta_1}{3L} = \frac{\delta_C}{2L}}$$

M=PL

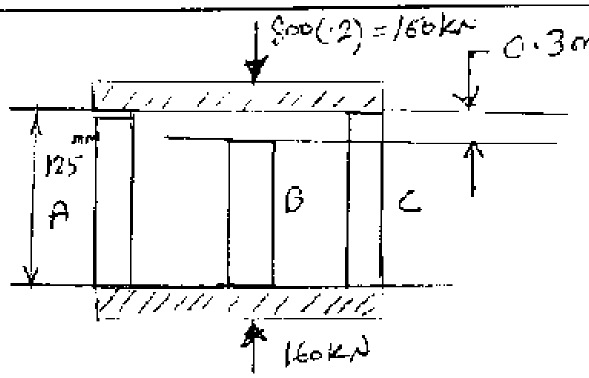


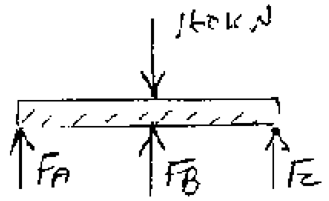
$$\boxed{\frac{\delta_1}{0.5} = \frac{\delta_2}{0.75}}$$

[illegible]

$E = 70 \times 10^9$ Pa

$A = 400$ mm^2





unknowns $\{F_A, F_B, F_C\}$

Equilibrium Equations:

① $\Sigma M_{@B} = 0$ ↺+ $\quad 100 F_C - F_A(100) = 0 \quad \Rightarrow \quad F_A = F_C = F$

② $\Sigma F_y = 0 \quad F_A + F_B + F_C = 160 \times 10^3$

$$\boxed{2F + F_B = 160 \times 10^3}$$

③ Compatability Equation: $\delta_A = \delta_C = \delta = \delta_B + 0.0003$

compatability equation should be expressed in terms of forces (unknowns).

$$\frac{F_A (L_A)}{A_A E_A} = \frac{F_B L_B}{A_B E_B} + 0.0003$$

$$\frac{F(0.125)}{(400 \times 10^{-6})(70 \times 10^9)} = \frac{F_B (0.1247)}{(400 \times 10^{-6})(70 \times 10^9)} + 0.0003$$

$$\boxed{125 F - 124.7 F_B = 8400}$$

solving Ⓐ & Ⓑ $\Rightarrow$ $F = 75762$ N

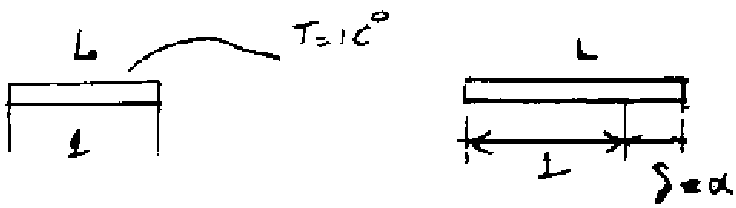
$F_B = 8547$ N

$$\sigma_A = \sigma_C = \sigma = \frac{F}{A} = \frac{75762}{400 \times 10^{-6}} = 189 \times 10^6 \text{ Pa} = 189 \text{ MPa}$$

$$\sigma_B = \frac{F_B}{A} = \frac{8547}{400 \times 10^{-6}} = 21.4 \times 10^6 \text{ Pa} = 21.4 \text{ MPa}$$

4.6 Thermal Stress

There are two types of stresses: one is called mechanical stress (due to physically applied load or reactions) and the other is thermal stresses for restrained members (due to increase or decrease in temperature)

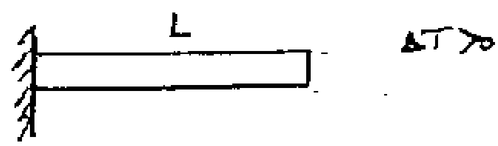


α = the change of unit length due to one degree of temp
= the strain due to one unit degree
= coefficient of thermal expansion $\left(\frac{m}{m}\frac{1}{°C}\right)$ or $\left(\frac{in}{in}\frac{1}{°F}\right)$

$\therefore$ the total strain due to an increase in temp ΔT

$$\epsilon_T = \alpha \Delta T \quad \text{(thermal strain)}$$

$$\delta = (\alpha \Delta T) L \quad \text{(change in length)}$$



$$\epsilon_{total} = \epsilon_{elastic} + \epsilon_{thermal}$$

$$\epsilon_{elastic} = \epsilon_{total} - \epsilon_{thermal}$$

Hooke's law $\sigma = E(\epsilon_{elastic})$

$$\epsilon_{thermal} = \alpha \Delta T L$$

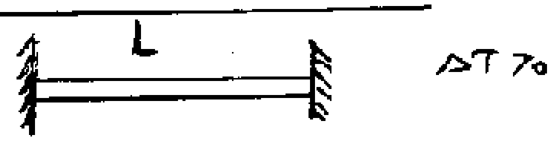
$$\epsilon_{total} = \epsilon_{thermal}$$

$$\epsilon_{elastic} = 0$$

$$\sigma = 0$$

$$\epsilon = \alpha \Delta T L$$

strain without stress



$$L\epsilon_{total} = L\epsilon_{thermal} + L\epsilon_{elastic}$$

$$\delta = \delta_{ther} + \delta_{elast}$$

$$0 = \alpha \Delta T L + \frac{NL}{EA}$$

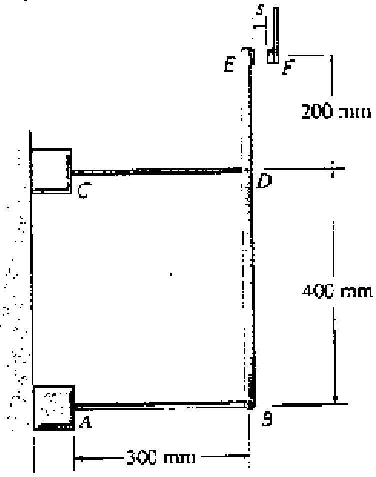
$$\Rightarrow \quad N = -\alpha \Delta T E A$$

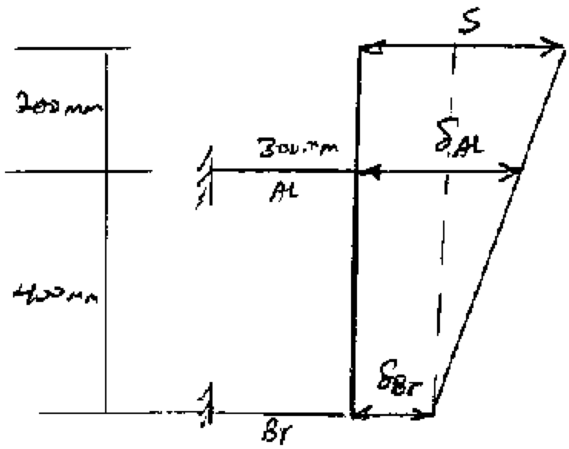
$$\sigma = \frac{N}{A} = -\alpha \Delta T E$$

$$\epsilon = \frac{\delta}{L} = 0$$

Stresses without strain

4-74. The electrical switch closes when the linkage rods CD and AB heat up, causing the rigid arm BDE both to translate and rotate until contact is made at F. Originally, BDE is vertical, and the temperature is 20°C. If AB is made of bronze C86100 and CD is made of aluminum 6061-T6, determine the gap s required so that the switch will close when the temperature becomes 110°C.





$= \frac{600}{400}$

$$s = \delta_{Br} + (\delta_{AL} - \delta_{Br})(1.5)$$

$$= -0.5\,\delta_{Br} + 1.5\,\delta_{AL}$$

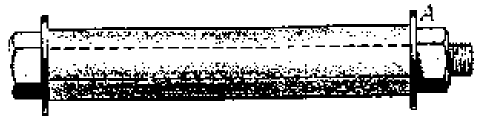
$$= -0.5\,\alpha_{Br}\,\Delta T\,L + 1.5\,\alpha_{AL}\,\Delta T\,L$$

$$= (-0.5\,\alpha_{Br} + 1.5\,\alpha_{AL})\,\Delta T\,L$$

$$= (-0.5 \times 17 \times 10^{-6} + 1.5 \times 24 \times 10^{-6})(90)(300)$$

$$= 0.7425 \text{ mm}$$

4–91. The steel bolt has a diameter of 7 mm and fits through an aluminum sleeve as shown. The sleeve has an inner diameter of 8 mm and an outer diameter of 10 mm. The nut at A is adjusted so that it just presses up against the sleeve. If the assembly is originally at a temperature of $T_1 = 20°C$ and then is heated to a temperature of $T_2 = 100°C$, determine the average normal stress in the bolt and the sleeve. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa, $\alpha_{st} = 14(10^{-6})/°C$, $\alpha_{al} = 23(10^{-6})/°C$.

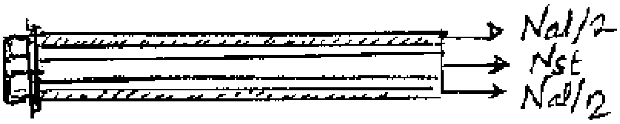


Prob. 4–91

$A_{al} = 28.3 \text{ mm}^2$ $\quad \Delta T = 80 \ C°$

$A_{st} = 38.5 \text{ mm}^2$ $\quad E_{al} = 70 \times 10^3$ MPa

$E_{st} = 200 \times 10^3$ MPa



$\Sigma F_x = 0 \qquad N_{st} + N_{al} = 0 \quad \Rightarrow \quad N_{st} = -N_{al} = N$

As $\alpha_{al} > \alpha_{st}$, then aluminum will not elongate freely as it will without presence of steel. Therefore aluminum will be in compression and steel will be in tension.

Compatability: $\qquad \delta_{al} = \delta_{st}$

$$\delta_{al}^{N_{al}} + \delta_{al}^{\Delta T} = \delta_{st}^{N_{st}} + \delta_{st}^{\Delta T}$$

$$\frac{N_{al}\, l}{E_{al} A_{al}} + \alpha_{al} \Delta T\, l = \frac{N_{st}\, l}{E_{st} A_{st}} + \alpha_{st} \Delta T\, l \qquad \begin{pmatrix} N_{al} = -N \\ N_{st} = N \end{pmatrix}$$

$$\frac{-N}{E_{al} A_{al}} - \frac{N}{E_{st} A_{st}} = \Delta T(\alpha_{st} - \alpha_{al}) \Rightarrow N = \frac{\Delta T(\alpha_{al} - \alpha_{st})}{\left(\frac{1}{E_{al} A_{al}} + \frac{1}{E_{st} A_{st}}\right)}$$

substituting $\Rightarrow \quad N = 1134.4\, N$

$$\sigma_{st} = \frac{N_{st}}{A_{st}} = \frac{1134.4}{\pi (49)^2/4} = 29.5 \text{ MPa}$$

$$\sigma_{al} = \frac{N_{al}}{A_{al}} = \frac{-1134.4}{\frac{\pi}{4}(100 - 64)} = 40.1 \text{ MPa}$$