Chapter 1.

Equilibrium Stress

Mechanics of Materials

The branch of mechanics that deals with the relationships between the external loads applied to the deformable body and the intensity of internal forces acting within the body.

body + external loads

- Internal forces (intensity of forces)
- deformation (change of shape & volume)
- stability (able to carry loads)

Mechanics of Solids

- * strength of materials (Mechanics of materials)
- * Theory of elasticity
- * Theory of plasticity
- * Fracture mechanics
- * Continuum damage mechanics

Equations of Equilibrium

Equations of Equilibrium. Equilibrium of a body requires both a ***balance of forces***, to prevent the body from translating or having accelerated motion along a straight or curved path, and a ***balance of moments***, to prevent the body from rotating. These conditions can be expressed mathematically by the two vector equations

$$\boxed{\begin{aligned} \Sigma \mathbf{F} &= \mathbf{0} \\ \Sigma \mathbf{M}_O &= \mathbf{0} \end{aligned}} \qquad (1\text{–}1)$$

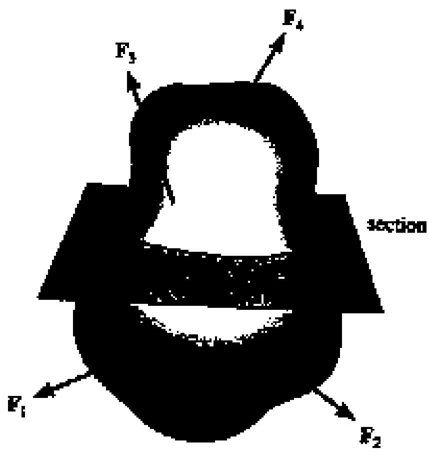
Here, $\Sigma \mathbf{F}$ represents the sum of all the forces acting on the body, and $\Sigma \mathbf{M}_O$ is the sum of the moments of all the forces about any point O either on or off the body. If an x, y, z coordinate system is established with the origin at point O, the force and moment vectors can be resolved into components along the coordinate axes and the above two equations can be written in scalar form as six equations, namely,

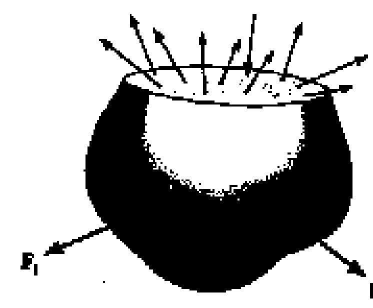
$$\boxed{\begin{array}{lll} \Sigma F_x = 0 & \Sigma F_y = 0 & \Sigma F_z = 0 \\ \Sigma M_x = 0 & \Sigma M_y = 0 & \Sigma M_z = 0 \end{array}} \qquad (1\text{–}2)$$

Often in engineering practice the loading on a body can be represented as a system of *coplanar forces*. If this is the case, and the forces lie in the x–y plane, then the conditions for equilibrium of the body can be specified by only three scalar equilibrium equations; that is,

$$\boxed{\begin{aligned} \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \\ \Sigma M_O &= 0 \end{aligned}} \qquad (1\text{–}3)$$

Three_ Dimentional Internal forces





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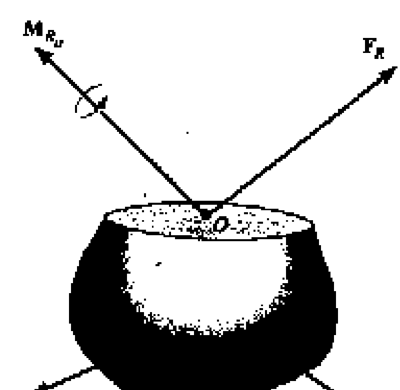

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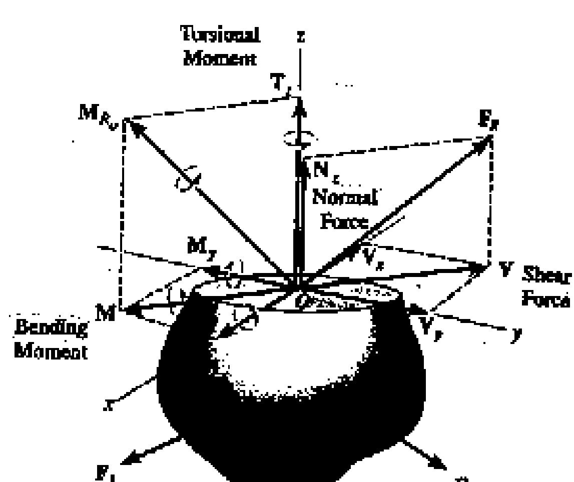


(d)

Fig. 1-2(c, d)

Two - Dimensional Internal Forces

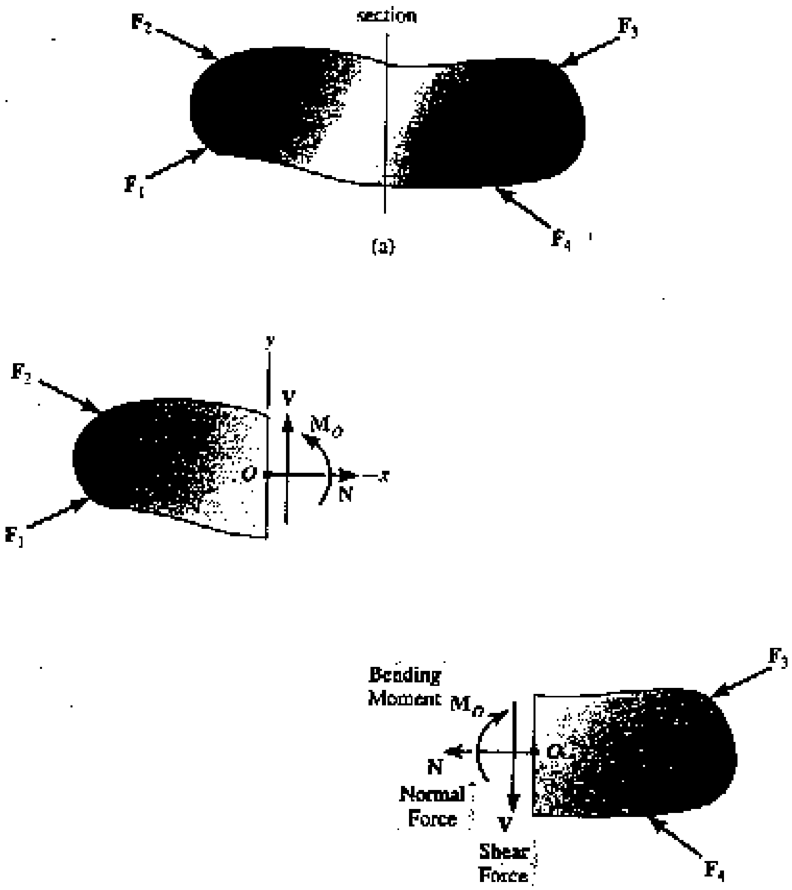
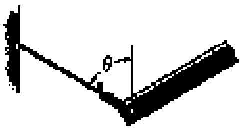
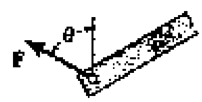
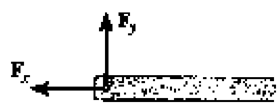
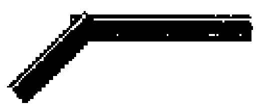
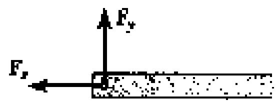
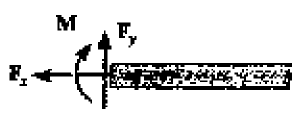


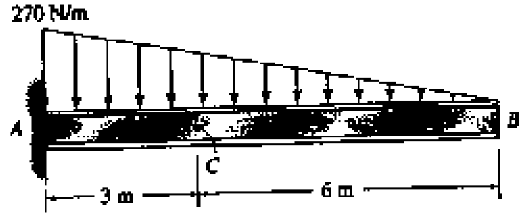
Fig. 1-3

Support Reactions

Type of connection	Reaction
 Cable	 One unknown: F
 Roller	 One unknown: F
 External pin	 Two unknowns: F_x, F_y
 Internal pin	 Two unknowns: F_x, F_y
 Fixed support	 Three unknowns: F_x, F_y, M

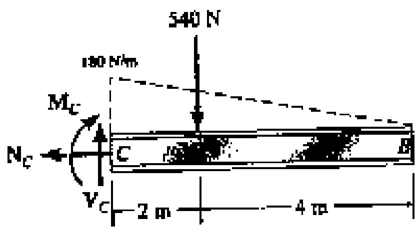
EXAMPLE 1-1

Determine the resultant internal loadings acting on the cross section at C of the beam shown in Fig. 1–4*a*.



(a)

Fig. 1–4



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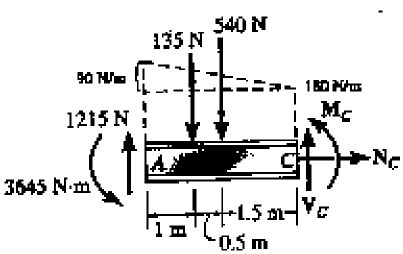


(c)

SOLUTION

Support Reactions. This problem can be solved in the most direct manner by considering segment CB of the beam, since then the support reactions at A do not have to be computed.

Free-Body Diagram. Passing an imaginary section perpendicular to the longitudinal axis of the beam yields the free-body diagram of segment CB shown in Fig. 1–4*b*. It is important to keep the distributed loading exactly where it is on the segment until *after* the section is made. Only then should this loading be replaced by a single resultant force. Notice that the intensity of the distributed loading at C is determined by proportion, i.e., from Fig. 1-4*a*, $w/6\text{ m} = 270\text{ N/m}/9\text{ m}$, $w = 180\text{ N/m}$. The magnitude of the distributive load is equal to the area under the loading curve (triangle) and acts through the centroid of this area. Thus, $F = \frac{1}{2}(180\text{ N/m})(6\text{ m}) = 540\text{ N}$, which acts $1/3(6\text{ m}) = 2\text{ m}$ from C as shown in Fig. 1–4*b*.

Equations of Equilibrium. Applying the equations of equilibrium we have

$\xrightarrow{+} \Sigma F_x = 0;$ $\qquad -N_C = 0$

$$N_C = 0 \qquad \textit{Ans.}$$

$+\uparrow \Sigma F_y = 0;$ $\qquad V_C - 540\text{ N} = 0$

$$V_C = 540\text{ N} \qquad \textit{Ans.}$$

$\circlearrowleft + \Sigma M_C = 0;$ $\qquad -M_C - 540\text{ N}(2\text{ m}) = 0$

$$M_C = -1080\text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

The negative sign indicates that M_C acts in the opposite direction on the free-body diagram. Try solving this problem using segment AC, by first obtaining the support reactions at A, which are given in Fig. 1–4c.

EXAMPLE 1–5

Determine the resultant internal loadings acting on the cross section at B of the pipe shown in Fig. 1–8a. The pipe has a mass of 2 kg/m and is subjected to both a vertical force of 50 N and a couple moment of 70 N · m at its end A. It is fixed to the wall at C.

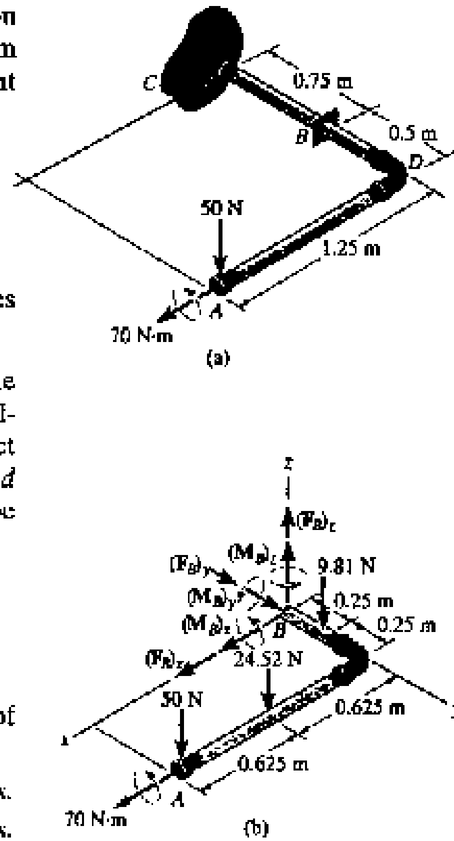


Fig. 1–8

SOLUTION

The problem can be solved by considering segment AB, which does *not* involve the support reactions at C.

Free-Body Diagram. The x, y, z axes are established at B and the free-body diagram of segment AB is shown in Fig. 1–8b. The resultant force and moment components at the section are assumed to act in the positive coordinate directions and to pass through the *centroid* of the cross-sectional area at B. The weight of each segment of pipe is calculated as follows:

$$W_{BD} = (2 \text{ kg/m})(0.5 \text{ m})(9.81 \text{ N/kg}) = 9.81 \text{ N}$$
$$W_{AD} = (2 \text{ kg/m})(1.25 \text{ m})(9.81 \text{ N/kg}) = 24.525 \text{ N}$$

These forces act through the center of gravity of each segment.

Equations of Equilibrium. Applying the six scalar equations of equilibrium, we have*

$\Sigma F_x = 0;\quad (F_B)_x = 0$ **Ans.**

$\Sigma F_y = 0;\quad (F_B)_y = 0$ **Ans.**

$\Sigma F_z = 0;\quad (F_B)_z - 9.81 \text{ N} - 24.525 \text{ N} - 50 \text{ N} = 0$

$(F_B)_z = 84.3 \text{ N}$ **Ans.**

$\Sigma(M_B)_x = 0;\quad (M_B)_x + 70 \text{ N}\cdot\text{m} - 50 \text{ N}(0.5 \text{ m}) - 24.525 \text{ N}(0.5 \text{ m}) - 9.81 \text{ N}(0.25 \text{ m}) = 0$

$(M_B)_x = -30.3 \text{ N}\cdot\text{m}$ **Ans.**

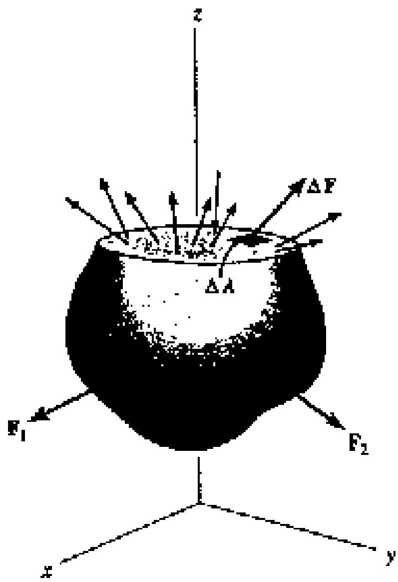
$\Sigma(M_B)_y = 0;\quad (M_B)_y + 24.525 \text{ N}(0.625 \text{ m}) + 50 \text{ N}(1.25 \text{ m}) = 0$

$(M_B)_y = -77.8 \text{ N}\cdot\text{m}$ **Ans.**

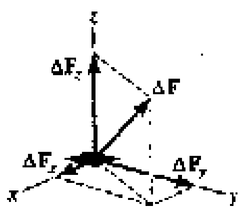
$\Sigma(M_B)_z = 0;\quad (M_B)_z = 0$ **Ans.**

What do the negative signs for $(M_B)_x$ and $(M_B)_y$ indicate? Note that the normal force $N_B = (F_B)_y = 0$, whereas the shear force is $V_B = \sqrt{(0)^2 + (84.3)^2} = 84.3$ N. Also, the torsional moment is $T_B = (M_B)_y = 77.8$ N · m and the bending moment is $M_B = \sqrt{(30.3)^2 + (0)^2} = 30.3$ N · m.

*The *magnitude* of each moment about an axis is equal to the magnitude of each force times the perpendicular distance from the axis to the line of action of the force. The *direction* of each moment is determined using the right-hand rule, with positive moments (thumb) directed along the positive coordinate axes.



(a)



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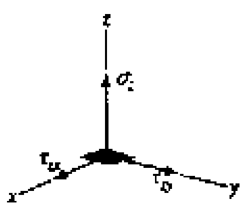


(c)

$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

and the two shear-stress components as

$$\tau_{zx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$

$$\tau_{zy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}$$

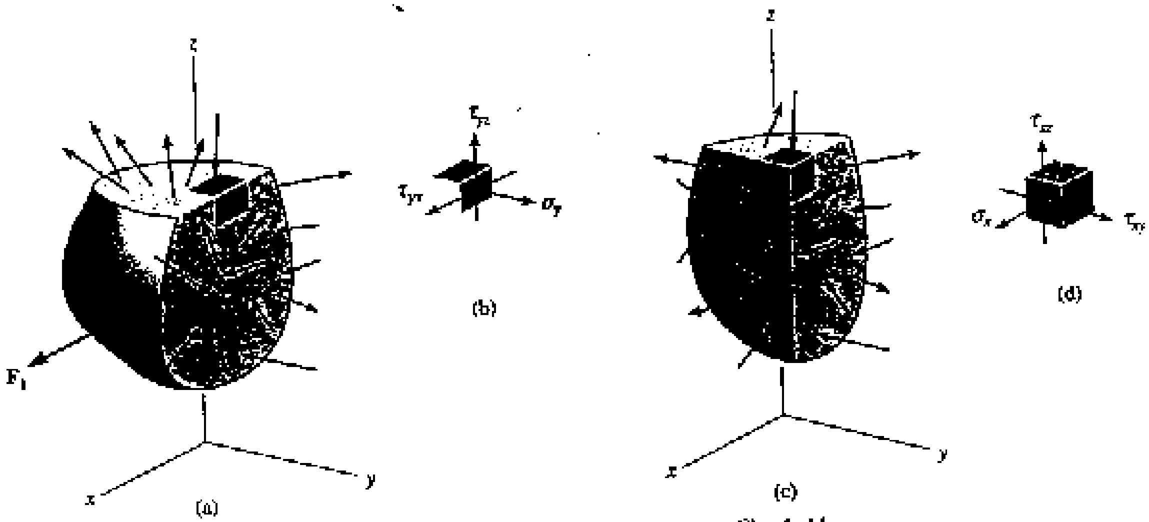
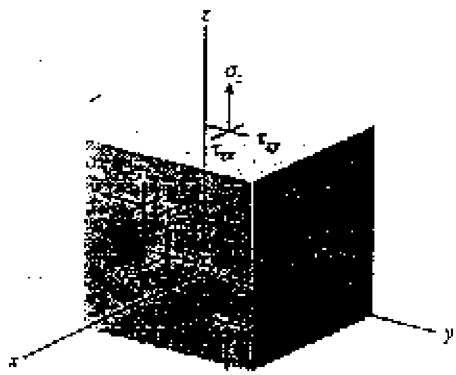


Fig. 1-11



General state of stress

Fig. 1-12

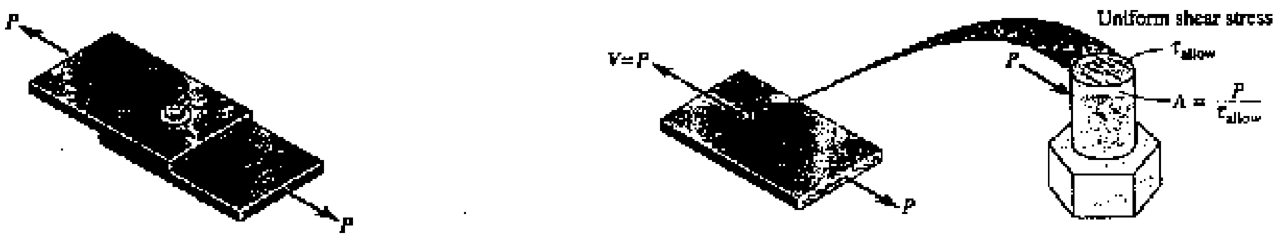
Types of stresses

1) Normal stress $\sigma = \dfrac{\text{Normal force}}{\text{Area}} = \dfrac{N}{A}$

2) Shear stress $\tau = \dfrac{\text{shear force}}{\text{Area}} = \dfrac{V}{A}$

3) Bearing stress $\sigma_B = \dfrac{\text{Force}}{\text{Area of contact}}$ (between two bodies

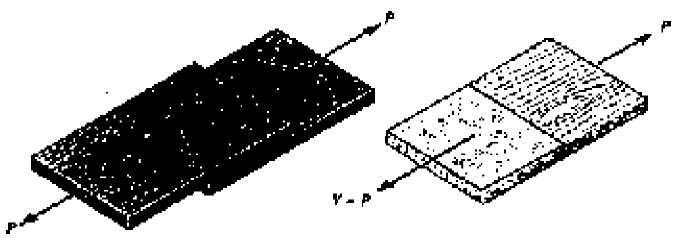
2-9) Shear stress in bolts and glue



(a) Fig. 1-28 (b) (c)

$$\tau_{bolt} = \frac{V_{bolt}}{A_{bolt}} = \frac{V_{bolt}}{\left(\frac{\pi d^2}{4}\right)}$$

d = diameter of bolt



(b) (c) (d)

Fig. 1-21

$$\boxed{\tau_{glue} = \frac{V_{glue}}{\text{glued area}}}$$

3) Bearing Stresses

$$\sigma_B = \frac{\text{Force between two bodies}}{\text{Area of contact}}$$

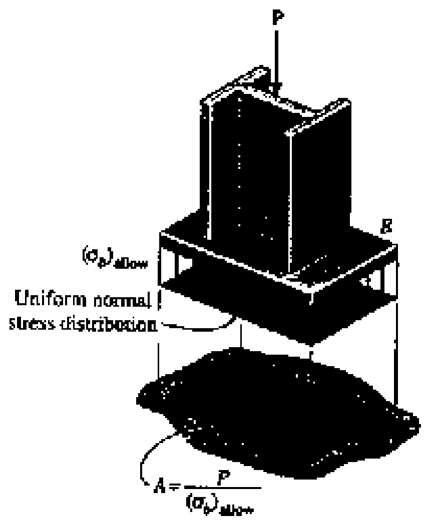
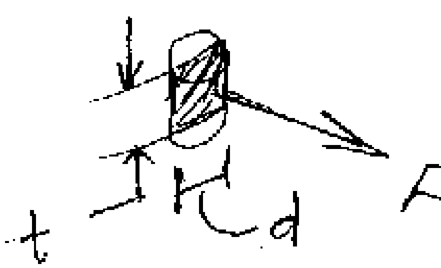


Fig. 1-29

$$(\sigma_B)_{bolt/plate} = \frac{\text{Force between bolt \& plate}}{\text{Projected Area} \perp \text{to loading}}$$

$$(\sigma_B)_{bolt} = \frac{F}{t\,d}$$



14 Normal Stress σ

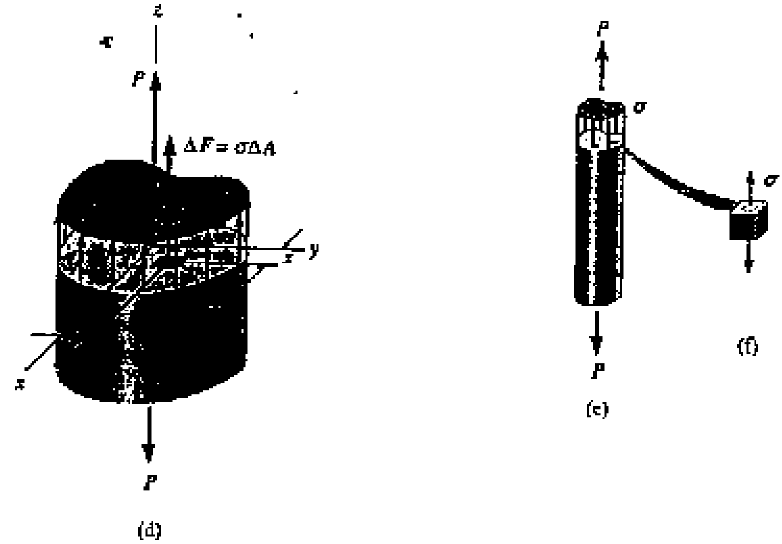


Fig. 1–14

Average Normal Stress Distribution. Assuming the bar is subjected to a constant uniform deformation as noted, then this deformation is caused by a *constant* normal stress σ, which is uniformly distributed over the bar's cross-sectional area, Fig. 1–14*d*. Since each area ΔA on the cross section is subjected to a force $\Delta F = \sigma \Delta A$, the *sum* of these forces acting over the entire cross-sectional area must then be equivalent to the internal force resultant **P** at the section. If we let $\Delta A \to dA$ and therefore $\Delta F \to dF$, then, recognizing σ is *constant*, we have

$+\uparrow F_{Rz} = \Sigma F_z;$ $$\int dF = \int_A \sigma \, dA$$
$$P = \sigma A$$

or

$$\boxed{\sigma = \frac{P}{A}} \quad (1\text{–}6)$$

Here

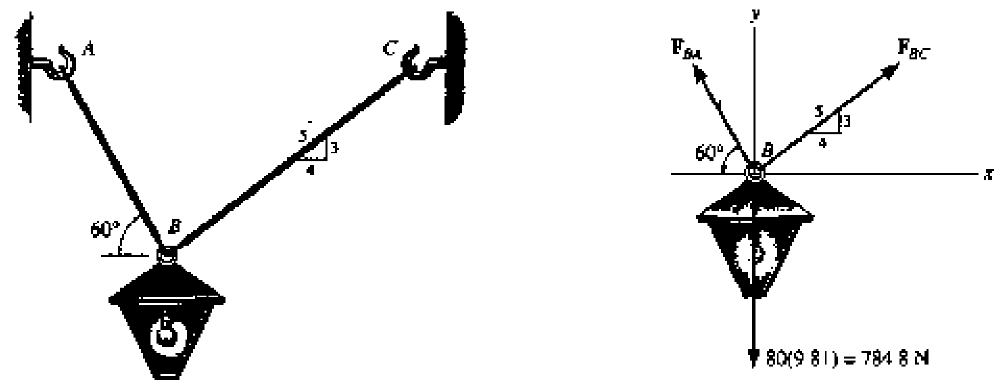
σ = average normal stress at any point on the cross-sectional area

P = internal resultant normal force, which is applied through the *centroid* of the cross-sectional area. P is determined using the method of sections and the equations of equilibrium.

A = cross-sectional area of the bar

EXAMPLE 1-7

The 80-kg lamp is supported by two rods AB and BC as shown in Fig. 1–17a. If AB has a diameter of 10 mm and BC has a diameter of 8 mm, determine which rod is subjected to the greater average normal stress.



(a) (b)

Fig. 1–17

SOLUTION

Internal Loading. We must first determine the axial force in each rod. A free-body diagram of the lamp is shown in Fig. 1–17b. Applying the equations of force equilibrium yields

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{BC}\left(\tfrac{4}{5}\right) - F_{BA}\cos 60^\circ = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC}\left(\tfrac{3}{5}\right) + F_{BA}\sin 60^\circ - 784.8\text{ N} = 0$$

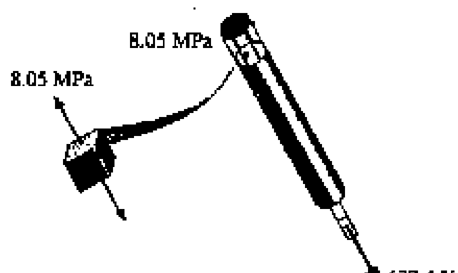
$$F_{BC} = 395.2\text{ N}, \qquad F_{BA} = 632.4\text{ N}$$

By Newton's third law of action, equal but opposite reaction, these forces subject the rods to tension throughout their length.

Average Normal Stress. Applying Eq. 1–6, we have

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{395.2\text{ N}}{\pi(0.004\text{ m})^2} = 7.86\text{ MPa} \qquad \textit{Ans.}$$

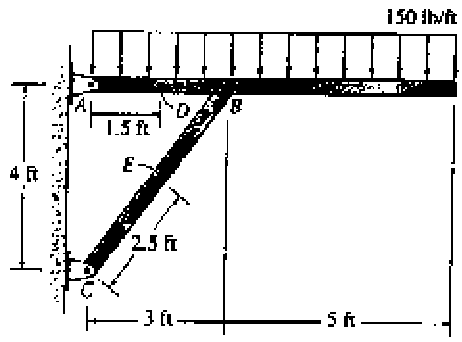
$$\sigma_{BA} = \frac{F_{BA}}{A_{BA}} = \frac{632.4\text{ N}}{\pi(0.005\text{ m})^2} = 8.05\text{ MPa}$$

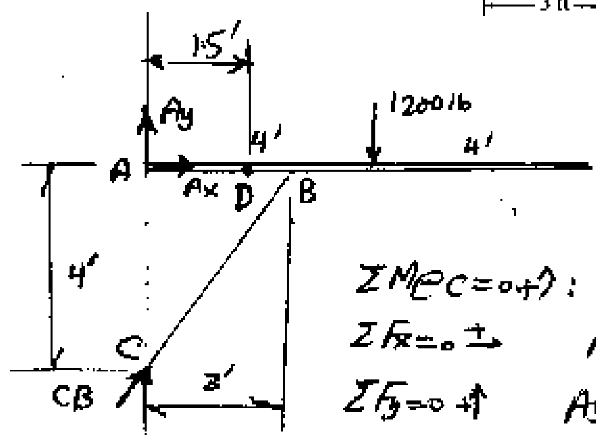


(d) (e)

The average normal stress distribution acting over a cross section of rod AB is shown in Fig. 1–17e, and at a point on this cross section, an element of material is stressed as shown in Fig. 1–17d.

1–19. Determine the resultant internal loadings on the cross sections through points D and E on the frame.



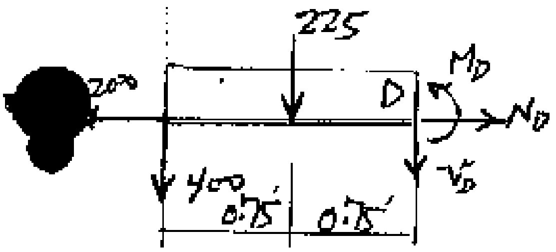


unknowns = {A_x, A_y, CB}

$\Sigma M@C = 0$: $-1200(4) - A_x(4) = 0 \Rightarrow A_x = -1200$ lb

$\Sigma F_x = 0$: $A_x + CB\left(\frac{3}{5}\right) = 0 \Rightarrow C_B = 2000$ lb

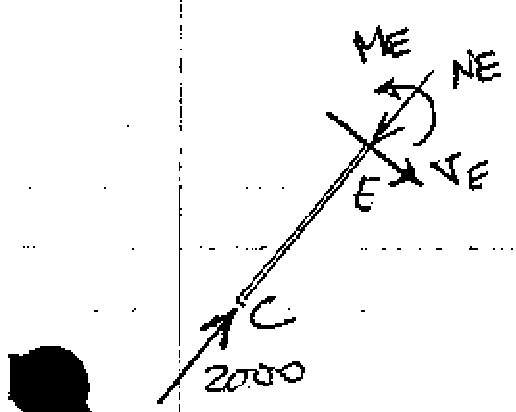
$\Sigma F_y = 0$: $A_y - 1200 + CB\left(\frac{4}{5}\right) = 0 \Rightarrow A_y = -400$ lb



$\Sigma F_x = 0$: $N_D - 1200 = 0$ $\quad N_D = 1200$ lb or $N_D = 1.2$ k

$\Sigma F_y = 0$: $-400 - 225 - V_D = 0$ $\quad V_D = -625$ lb or $V_D = 0.625$ k

$\Sigma M@D = 0$: $M_D + 225(0.75) + 400(1.5) = 0$ $\quad M_D = -769$ lb-ft

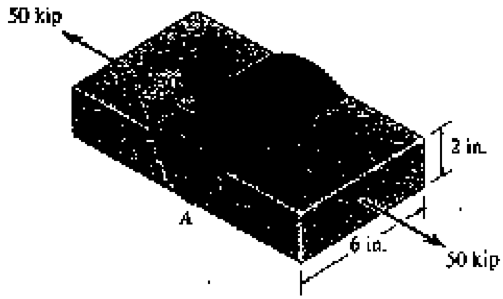


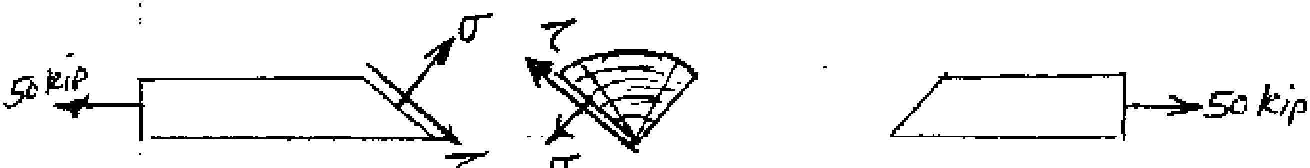
$\Sigma F_y = 0$ $\quad V_E = 0$

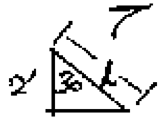
$\Sigma F_x = 0$ $\quad N_E = 2000$ lb

$\Sigma M@E = 0$ $\quad M_E = 0$

1–49. The open square butt joint is used to transmit a force of 50 kip from one plate to the other. Determine the average normal and average shear stress components that this loading creates on the face of the weld, section AB.

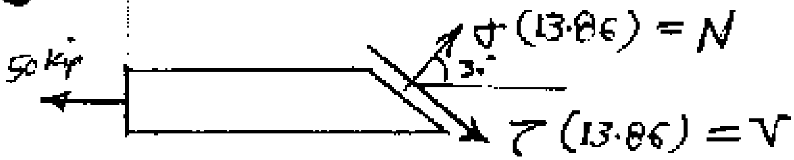






$$l \cos 30 = 2 \implies l = \frac{2}{\cos 30} = 2.31$$

area where σ and τ are acting $A = (2.31)(6) = 13.86\ in^2$

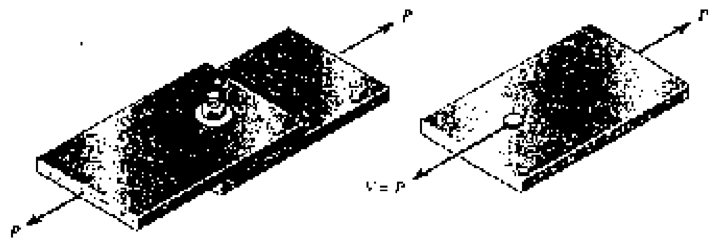


$$\Sigma F_y = 0 \quad \sigma(13.86)\sin 30 - \tau(13.86)\cos 30 = 0 \quad (1)$$
$$\Sigma F_x = 0 \quad \sigma(13.86)\cos 30 + \tau(13.86)\sin 30 - 50\times 10^3 = 0 \quad (2)$$

Solving (1) & (2) $\implies$ $\sigma = 3.125$ ksi

$\tau = 1.8$ ksi

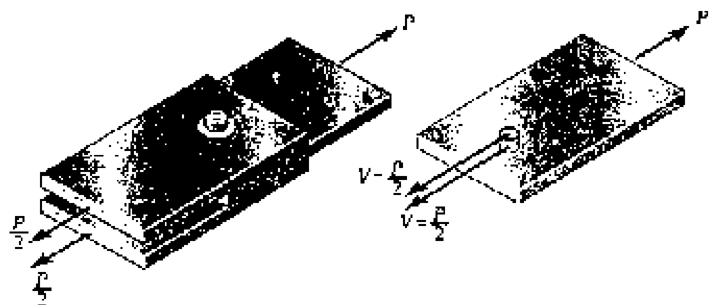
2-b) Single shear versus double shear Connections



(a)

single shear connections

$$\tau = \frac{V}{A_{bolt}} = \frac{P}{A_{bolt}}$$

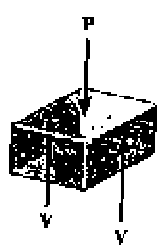


(a)

double shear connections

$$\tau = \frac{V}{A_{bolt}} = \frac{P/2}{A_{bolt}}$$

2-c) shear stress in beams



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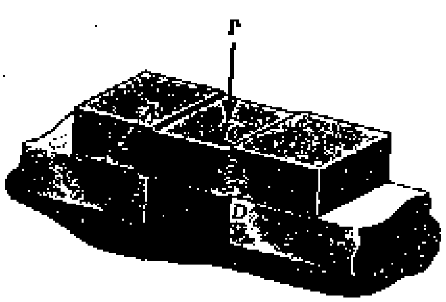
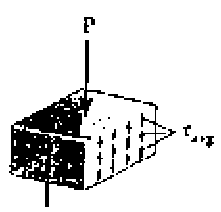
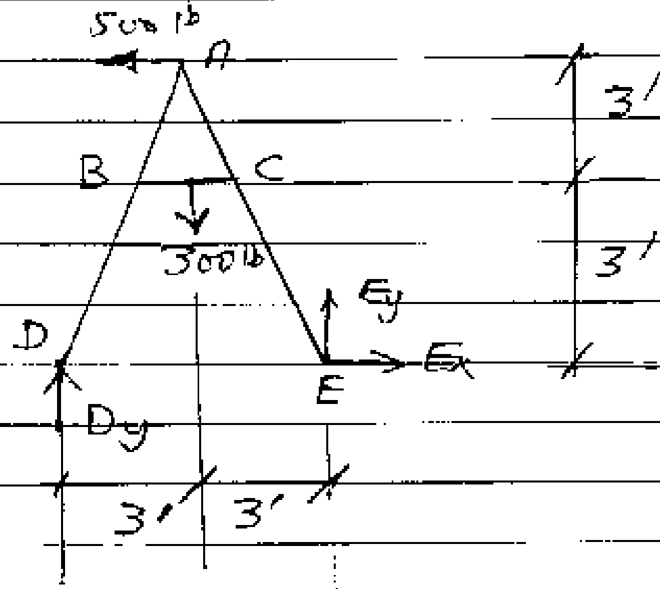


(a)

$$\boxed{\tau_{avg} = \frac{V}{A}}$$



(c)



Find shear stress in pins E & D if both are double shear and their diameter = 0.25 in

STEP ① . Finding reactions

$\sum M @ D = 0 + \uparrow \quad -300(3) + 500(6) + E_y(6) = 0$

$\Rightarrow$

$\sum F_y = 0 \uparrow \quad D_y + E_y - 300 = 0 \Rightarrow$

$\sum F_x = 0 \quad E_x = 500$

$\boxed{\begin{array}{l} E_y = 350 \text{ lb} \\ D_y = 650 \text{ lb} \\ E_x = 500 \text{ lb} \end{array}}$

Reaction at pin E : $R = \sqrt{E_x^2 + E_y^2}$

$= \sqrt{(350)^2 + (500)^2} = 610.3 \text{ lb}$

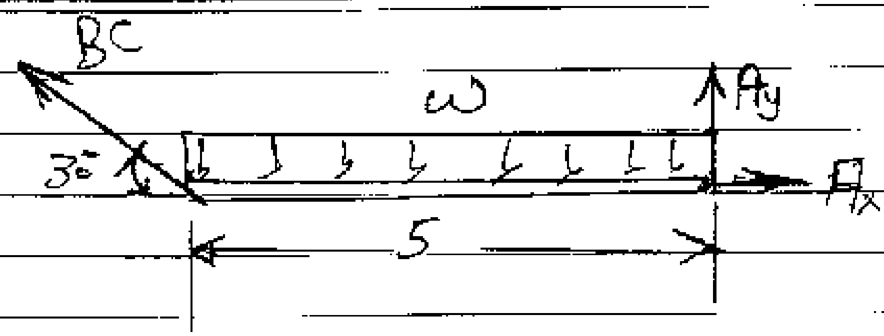
shear at pin E $= V_E = \frac{R}{2} = 305.2 \text{ lb}$ (double shear)

shear at pin D $= \frac{R_D}{2} = \frac{D_y}{2} = 325 \text{ lb}$ (double shear)

$\tau_E = \frac{V_E}{A_{pin E}} = \frac{305.2 \text{ lb}}{\frac{\pi (d^2)}{4}} = \frac{305.2}{\frac{\pi}{4}(.25)^2} = \underline{\underline{6.2 \text{ ksi}}}$

$\tau_D = \frac{V_D}{A_{pin D}} = \frac{325}{\frac{\pi (d^2)}{4}} = \frac{325}{\frac{\pi}{4}(0.25)^2} = \underline{\underline{6.62 \text{ ksi}}}$

1[illegible]8
42



Find w_{max} if $\tau_{all} = 80\,MPa$ for Pin A

$$\sum M_{(@A=0)}\quad 5\omega(2.5) - BC\sin 30\,(5) = 0$$
$$\Rightarrow \quad BC = 5\omega$$

$$\tau_c = \frac{V_c}{A_{pin}} = \frac{5\omega/2}{\frac{\pi(18)^2}{4}} = 80\,MPa$$

$$\Rightarrow \quad \omega = 8143\,N$$

$$\sum F_x \Rightarrow \quad A_x = 5\omega\cos 30 \Rightarrow \quad A_x = 4.33\omega$$

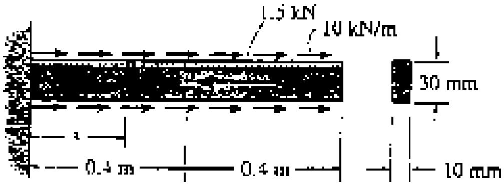
$$\sum F_y = 0 \quad BC\sin 30 - \omega(5) + A_y = 0 \qquad A_y = 2.5\omega$$

$$R_{pin\,A} = \sqrt{[4.33\omega]^2 + [2.5\omega]^2} = 5\omega$$

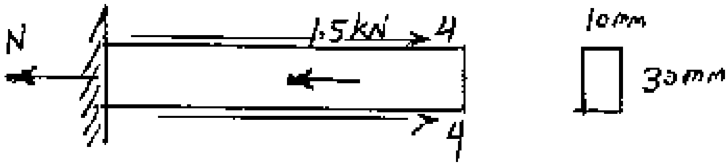
$$\tau_A = \frac{5\omega/2}{\frac{\pi(18)^2}{4}} = 80\,MPa \quad \Rightarrow \quad \omega = 8143\,N$$

$$\therefore \quad \omega = 8.143\,kN$$

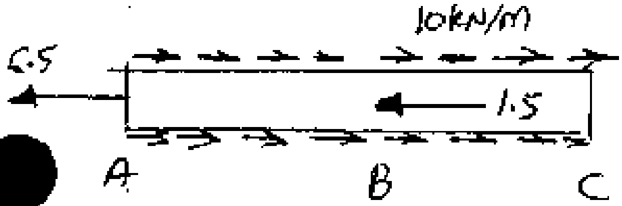
1–62. The bar is subjected to a uniform distributed axial loading of 10 kN/m and a concentrated force of 1.5 kN at its midpoint as shown. Determine the maximum average normal stress in the bar and its location x.



Prob. 1–62



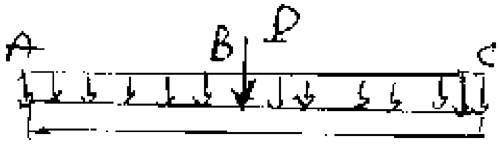
$\Sigma F_x = 0 \quad (10)(.8) - 1.5 - N = 0 \Rightarrow N = 6.5\text{ kN}$



here 1.5 changes function of axial force N

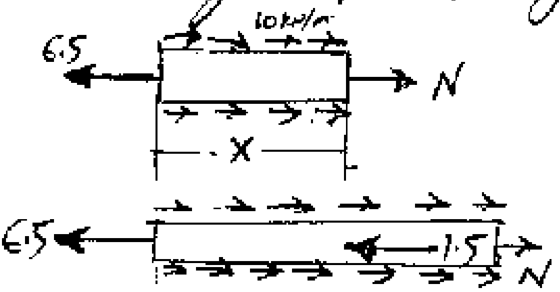
need two functions of N

Segment AB & Segment BC



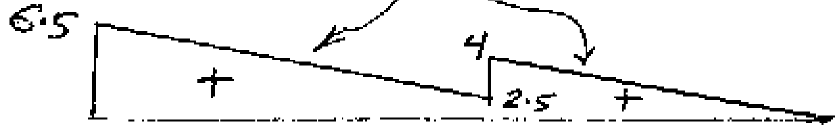
P changes function of V

need two functions of shear (Seg. AB & Seg. BC)



$\Sigma F_x = 0 \qquad N = 6.5 - 10x \qquad 0 \le x \le 0.4$

$\Sigma F_x = 0 \qquad N = 8 - 10x \qquad 0.4 < x \le 0.8$



Normal force N diagram.

∴ Maximum normal stress at maximum normal force at $x = 0$, $N = 6.5$

$$\therefore \sigma = \frac{N}{A} = \frac{6.5 \times 10^3}{(10)(30)} = 21.67\text{ MPa}$$

1.6 Allowable Stress

Q) First of all, why do we need to calculate stresses?

A) we need to calculate stresses (σ & τ) for two major tasks:

1) To do analysis of existing or proposed structure or machine

2) To design of a new structure or machine

ultimate stress σ_U is calculated by deviding the ultimate load P_u by the original area

$$\sigma_U = \frac{P_u}{A_o}$$

In actual structure, we do not allow the load to reach to P_u and it has to be far away from P_u for safty. The ratio of the actual load to ultimate load is called factor of safty

$$F.S. = \frac{P_{ultimate}}{P_{actual}}$$

$$F.S. = \frac{\sigma_{ultimate}}{\sigma_{allowable}}$$

we need factor of safty due to the approximate method of calculating stresses, and defects which material may have during fabrication. Also due to deterioration and corrosion which may occur in future

$$2 \leq F.S \leq 4$$ depends on risk

There are two key words in the engineering mechanics :

① Investigation (or analysis) of an existing structure in which both loads and geometry are given and you are requested to check for its safty :

$$\sigma_{cal.} \leq \sigma_{all}$$

$$\tau_{cal.} \leq \tau_{all}$$

Always the calculated stress (using formula) should be less than or equal the allowable for a structure to be safe.

② Design of a proposed structure in which loads are given but not the dimension of the structure.

$$\sigma_{cal.} = \sigma_{all}$$

$$\tau_{cal.} = \tau_{all}$$

From these equations find the unknown (area or diameter or length).

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$$\tau_{cal.} = \tau_{all}$$

From these equations find the unknown (area or diameter or length).

1.7 Design of Simple Connections

By making simplifying assumptions regarding the behavior of the material, the equations $\sigma = P/A$ and $\tau_{avg} = V/A$ can often be used to analyze or design a simple connection or a mechanical element. In particular, if a member is subjected to a *normal force* at a section, its required area at the section is determined from

$$A = \frac{P}{\sigma_{allow}} \tag{1-11}$$

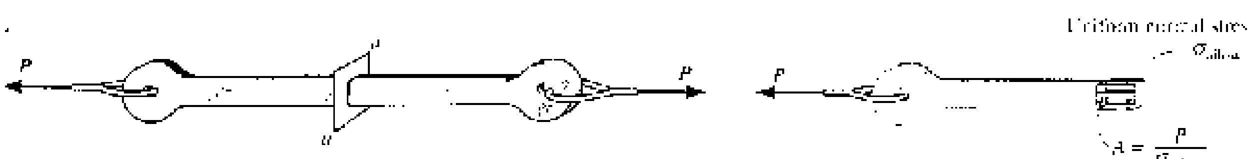
On the other hand, if the section is subjected to a *shear force*, then the required area at the section is

$$A = \frac{V}{\tau_{allow}} \tag{1-12}$$

As discussed in Sec. 1.6, the allowable stress used in each of these equations is determined either by applying a factor of safety to a specified normal or shear stress or by finding these stresses directly from an appropriate design code.

We will now discuss four common types of problems for which the above equations can be used for design.

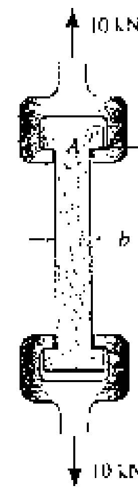
(1) Normal stress



(a) Fig. 1–27 (b)

94

1–94. The wood specimen is subjected to the pull of 10 kN in a tension testing machine. If the allowable normal stress for the wood is $(\sigma_t)_{allow} = 12$ MPa and the allowable shear stress is $\tau_{allow} = 1.2$ MPa, determine the required dimensions b and t so that the specimen reaches these stresses simultaneously. The specimen has a width of 25 mm.



Prob. 1–94

① $\sigma_{cal} = \sigma_{all}$

$$\frac{N}{A} = 12 \text{ MPa}$$

$$\frac{10 \times 10^3}{(b)(25)} = 12 \text{ MPa} \Rightarrow b = 33.33 \text{ mm}$$

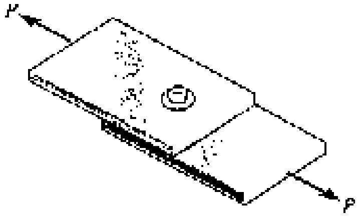
② $\tau_{cal} = \tau_{all}$

$$\frac{V}{A} = 1.2 \text{ MPa}$$

$$\frac{10 \times 10^3}{2(t)(25)} = 1.2 \text{ MPa} \Rightarrow t = 166.6 \text{ mm}$$

③ How much "a" if $(\sigma_b)_{all} = 15$ MPa ?

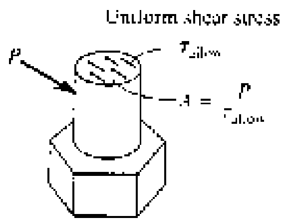
② Shear stress



(a)

Fig. 1–28

(b)



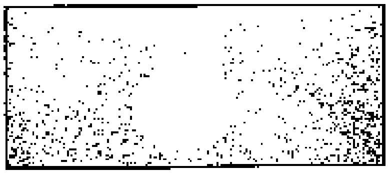
(c)

① $\sigma_{cal}^{plate} < \sigma_{all}^{plate}$

② $\tau_{cal}^{pin} < \tau_{all}^{pin}$

③ $(\sigma_b)_{cal}^{plate-pin} < (\sigma_b)_{all}^{plate-pin}$

③ Bearing stress



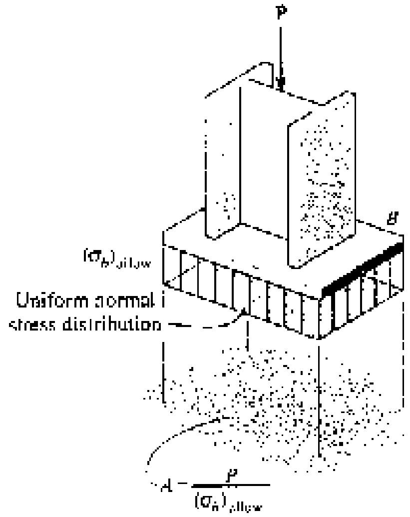
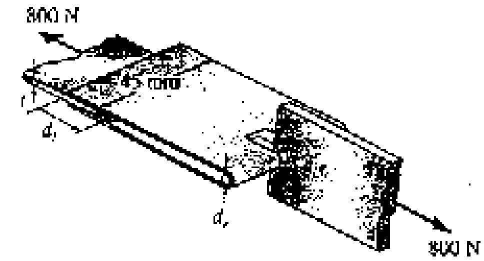


Fig. 1–29

1–98. The lapbelt assembly is to be subjected to a force of 800 N. Determine (a) the required thickness t of the belt if the allowable tensile stress for the material is $(\sigma_t)_{allow} = 10$ MPa, (b) the required lap length d_l if the glue can sustain an allowable shear stress of $(\tau_{allow})_g = 0.75$ MPa, and (c) the required diameter d_r of the pin if the allowable shear stress for the pin is $(\tau_{allow})_p = 30$ MPa.



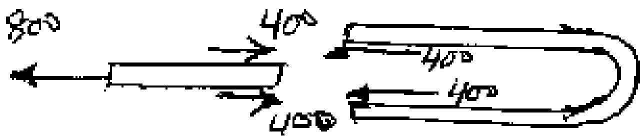
Remember that $\sigma_{calculated} \leq \sigma_{allowable}$

$(\sigma_{cal.} \leq \sigma_{all})$ or $(\tau_{cal.} \leq \tau_{all})$

(a) $\sigma_{cal.} = \sigma_{all}$

$$\frac{800}{45 \cdot (t)} = 10 \text{ MPa} \Rightarrow \boxed{t = 1.78 \text{ mm}}$$

(b)



$\tau_{cal.} = \tau_{all}$

$$\frac{400}{(45)\,d_l} = 0.75 \Rightarrow \boxed{d_l = 11.85 \text{ mm}}$$

(c)

$\tau_{cal} = \tau_{all}$

$$\frac{400}{\frac{\pi d_r^2}{4}} = 30 \Rightarrow \boxed{d_r = 4.12 \text{ mm}}$$