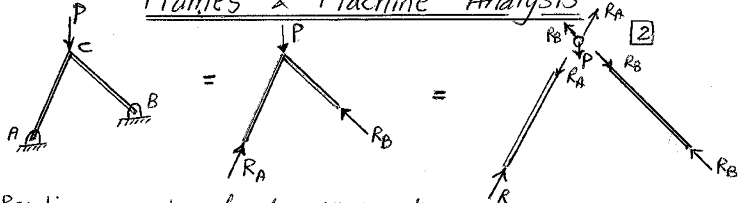
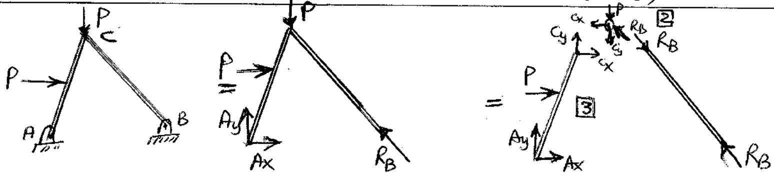
Frames & Machine Analysis

1



- Reactions can be found without disconnecting members
- since all members are two-force members (links) then it is TRUSS

2



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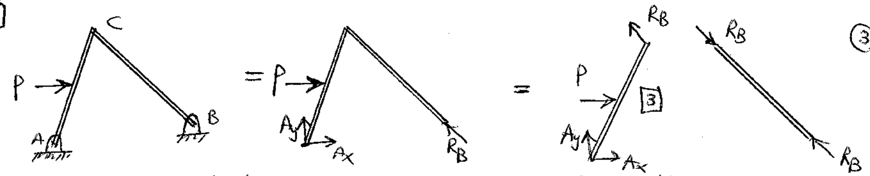
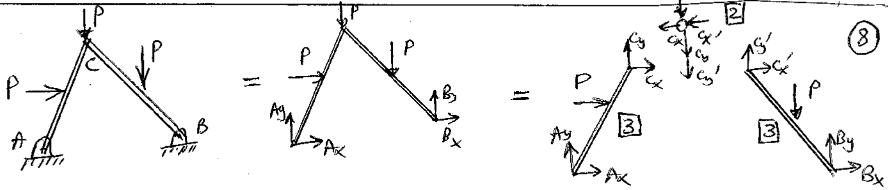


- No need to consider equilibrium of joint
- Start lable two-force members first. Then carry their opposite sense in the other members.

4



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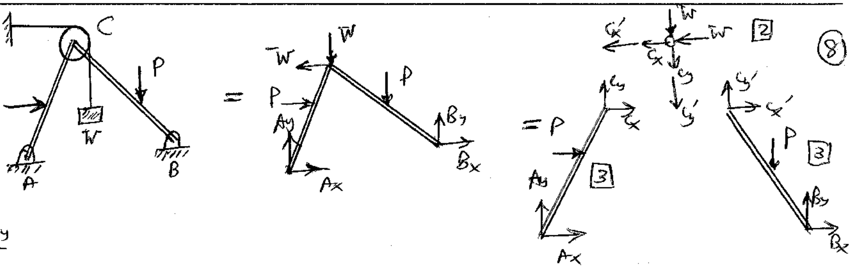
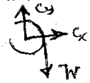


F.B.D of pully



$C_x = W$

$C_y = W$

○ ——— # of unknowns

□ ——— # of equations of equilibrium.

6-91
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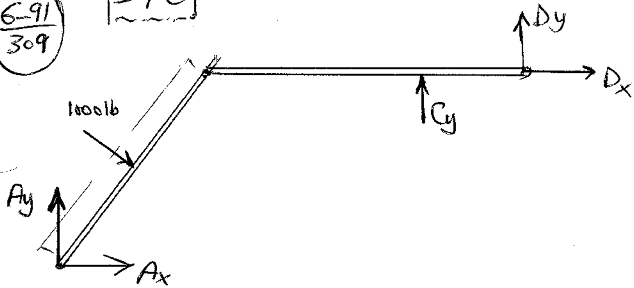
step ①



step ②

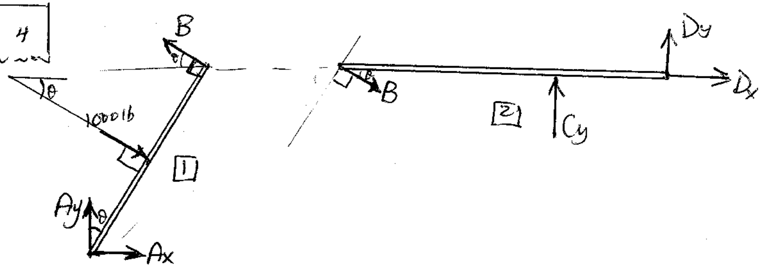
# of external reactions $\{A_x, A_y, C_y, D_x, D_y\} = 5$
# of equations of equilibrium $= 3$

∴ Reactions > Equations
∴ Need to disconnect members.

Step ③

No pully

step 4



total equations of equilibrium $= 3 + 3 = 6$
total number of unknowns $= 6$ $(A_x, A_y, B, C_y, D_x, D_y)$

∴ # of equations = # of unknowns

start to apply equations of equilibrium.

Step 5

Member $\boxed{1}$

$$\Sigma M_{@A} = 0 \; +\circlearrowleft : \; -1000\,(2.5) + B(5) = 0 \;\Rightarrow\; B = 500 \text{ lb}$$

$$\Sigma F_x = 0 \; +\rightarrow : \; A_x + 1000\left(\tfrac{3}{5}\right) \mp B\left(\tfrac{3}{5}\right) = 0 \;\Rightarrow\; A_x = -300 \text{ lb}$$

$$\Sigma F_y = 0 \; +\uparrow : \; A_y - 1000\left(\tfrac{4}{5}\right) + B\left(\tfrac{4}{5}\right) = 0 \;\Rightarrow\; A_y = 400 \text{ lb}$$

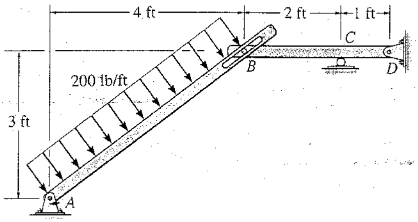
Member $\boxed{2}$

$$\Sigma M_{@D} = 0 \; +\circlearrowleft : \; -C_y\,(1) + B\left(\tfrac{4}{5}\right)(3) = 0 \;\Rightarrow\; C_y = +1200 \text{ lb}$$

$$\Sigma F_x = 0 \; +\rightarrow : \; D_x + B\left(\tfrac{3}{5}\right) = 0 \;\Rightarrow\; D_x = -300 \text{ lb}$$

$$\Sigma F_y = 0 \; +\uparrow : \; -B\left(\tfrac{4}{5}\right) + C_y + D_y = 0 \;\Rightarrow\; D_y = -800 \text{ lb}$$

6-91. Determine the reactions at the supports. The pin, attached to member *BCD*, passes through a smooth slot in member *AB*.



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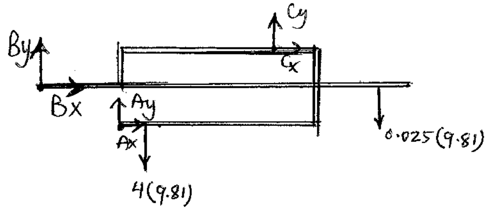
(1)

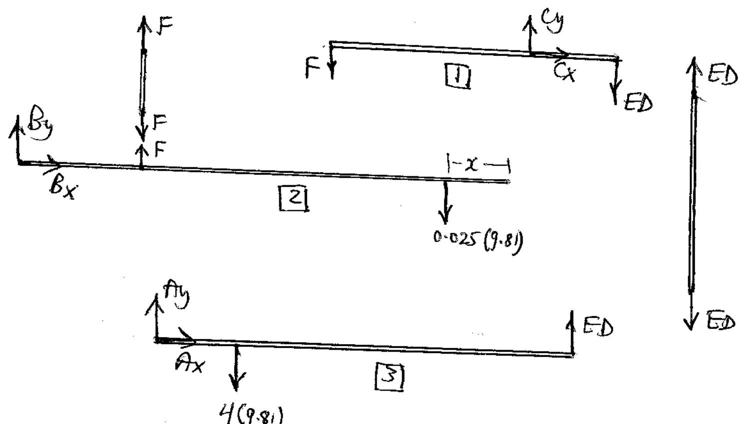


(2) # of external reactions : {Bx, By, Ax, Ay, Cx, Cyx} = 7
# of equations of equilibrium = 3

∴ Reactions > Equations
∴ Need to disconnect members.

(3) No pully

(4)



# of unknowns : {Ax, Ay, Bx, By, Cx, Cy, F, ED, x} = 9
# of equations of equilibrium = 3 + 3 + 3 = 9
∴ # of equations = # of unknowns

ok.

Step 5

Member [3]

$\sum M@A=0 +\curvearrowleft$ $\quad ED(375)-4(9.81)(50)=0 \Rightarrow \quad ED=5.23\,N$

$\sum F_x=0 \quad A_x=0$

$\sum F_y=0 \quad A_y-4(9.81)+ED=0 \Rightarrow \quad A_y=34.01\,N$

Member [1]

$\sum F_x=0 \quad C_x=0$

$\sum M@F=0 +\curvearrowleft$ $\quad -ED(375)+C_y(300)=0 \Rightarrow \quad C_y=6.54N$

$\sum F_y=0 \quad -F+C_y-ED=0 \Rightarrow \quad F=1.31\,N$

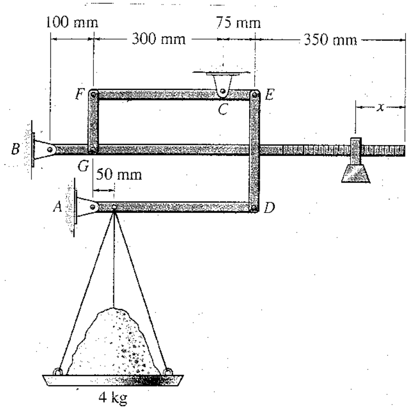
Member [2]

$\sum F_y=0 \quad B_y+F-0.025(9.81)=0 \Rightarrow \quad B_y=-1.06\,N$

$\sum F_x=0 \quad B_x=0$

$\sum M@B=0 +\curvearrowleft$ $\quad F(100)-0.025(9.81)(825-x)=0 \Rightarrow \quad x=291\text{ mm.}$

6-97. The compound arrangement of the pan scale is shown. If the mass on the pan is 4 kg, determine the horizontal and vertical components at pins A, B, and C and the distance x of the 25-g mass to keep the scale in balance.



6-119 / 315

① By inspection

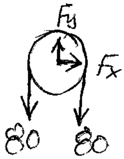
② # of external reactions $= 4 \; \{A_x, A_y, E_x, E_y\}$

# of equations $= 3 \; \{\Sigma F_x = 0, \; \Sigma F_y = 0, \; \Sigma M = 0\}$

$\therefore$ # Reactions $>$ # of equations

$\therefore$ Need to disconnect members.

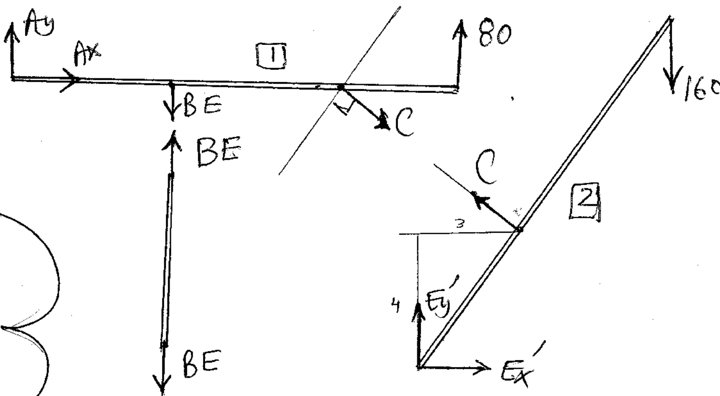
③



$\Sigma F_x = 0 \qquad F_x = 0$

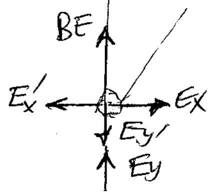
$\Sigma F_y = 0 \qquad F_y = 160 \uparrow$

④



Two members and one joint

load applied at the joint



1	C	$\Sigma M @ E$ [2]
3	A_x	$\Sigma F_x = 0$ [1]
4	A_y	$\Sigma F_y = 0$ [1]
2	BE	$\Sigma M @ A$ [1]

step 5

Member (2)

$\Sigma M@E = 0 \; +\circlearrowleft \quad -160(6) + C(5) = 0 \Rightarrow C = 192$

$\therefore \quad C_x = \frac{4}{5}(C) = 153.6\ N \leftarrow$

$C_y = \frac{3}{5}(C) = 115.2\ N \uparrow$

Member (1)

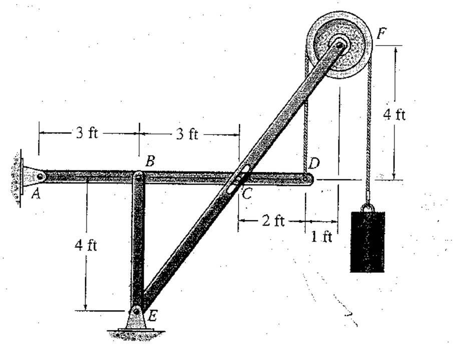
$\Sigma M@A = 0 \; +\circlearrowleft \quad 80(8) - 115.2(6) - BE(3) = 0 \Rightarrow BE = \frac{-51.2}{3} N$

$BE = -17.1\ N$

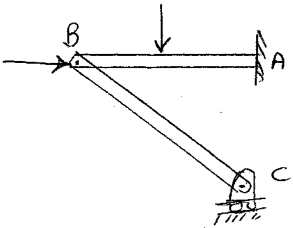
$\Sigma F_x = 0 \quad A_x + 153.6 = 0 \Rightarrow A_x = -153.6\ N$

$\Sigma F_y = 0 \quad A_y - BE - 115.2 + 80 = 0 \quad A_y = 18.1\ N$

6-119. The frame supports the 80-lb weight. Determine the horizontal and vertical components of force which the pins exert on member *ABCD*. Note that the pin at *C* is attached to member *ABCD* and passes through the smooth slot in member *ECF*.



Ch. 7 Internal Forces

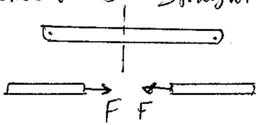


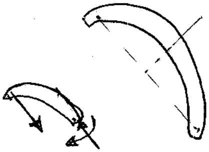
① Need to find reactions at supports A and C so as to avoid failure at these supports.

② Need to find forces in pin B so as to aviod failure at the pin B

③ Need to find forces within members BA and member BC so as to avoid failure of the member itself. Therefore one needs to find the internal forces and moment.

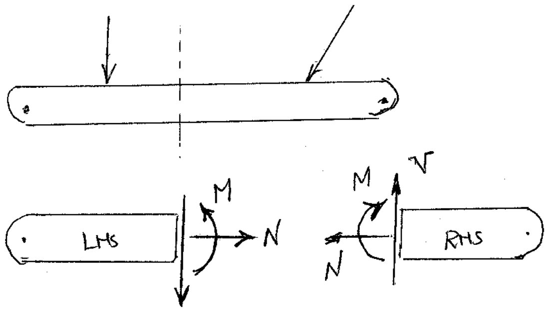
@ For two-force member, (as truss member or member on frame connected by two pins at the ends with no load acting on it), there is only one internal force and is called axial force. if the member is straight. otherwise it will develope moment.





(b) None-two force member will devlope three unknows.

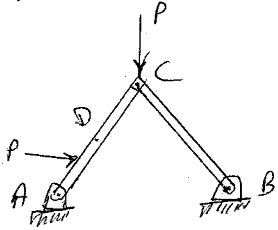
- Shear force ⊥ the axis V
- Normal force // the axis N
- Moment M



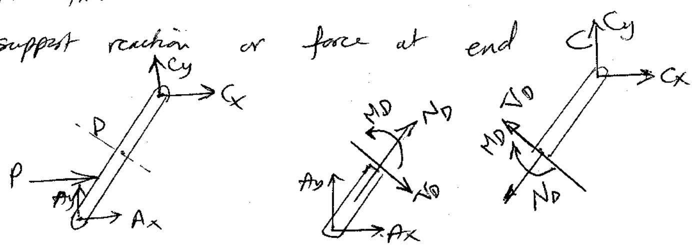
RHS — right hand side part

LHS — left hand side part

For frame member, one needs to find the forces at one end before cutting (or sectioning) the member



To find internal force at D (one needs to find support reaction or force at end C



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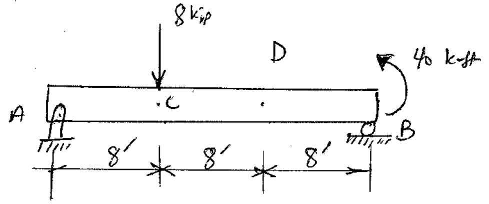
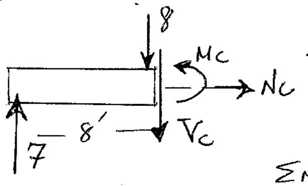


You have the choice either finding reaction at support A or reaction at support B. Let's find reaction at A

$\sum M_{@B} = 0 +\circlearrowleft$ $\quad 40 + 8(16) - A_y(24) = 0$ $\quad A_y = 7$

$\boxed{A_x = 0}$

Internal forces at C



$\sum F_y = 0: \quad 7 - 8 - V_c = 0$

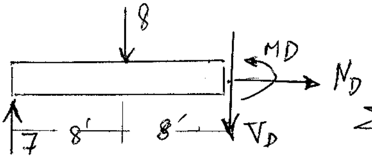
$\boxed{V_c = -1}$ kip

$\sum M_{@C} = 0 +\circlearrowleft \quad M_c - 7(8) = 0$

$\boxed{M_C = 42 \text{ kip-ft}}$

$\sum F_x = 0 \quad \boxed{N_C = 0}$

Internal forces at D



$\sum F_y = 0 \quad \boxed{V_D = -1}$

$\sum M_{@D} = 0 +\circlearrowleft$

$M_D + 8(8) - 7(16) = 0$

$\boxed{M_D = 48 \text{ kip-ft}}$

$\sum F_x = 0 \quad \boxed{N_D = 0}$

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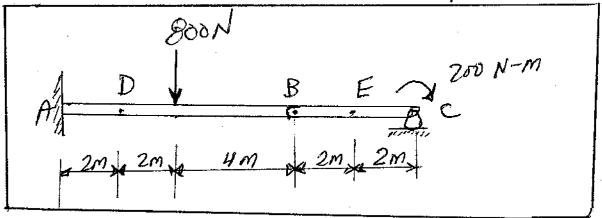
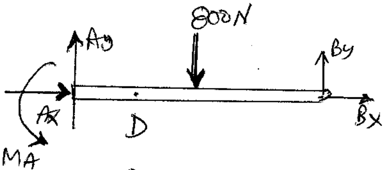
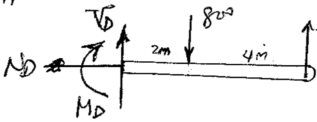
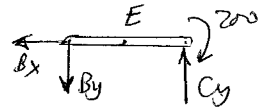








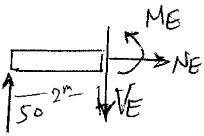
$\Sigma M @ C = 0 \circlearrowleft +$ $\quad 4B_y - 200 = 0$

$\boxed{B_y = 50}$

$\Sigma F_x = 0$ $\quad \boxed{B_x = 0}$

$\boxed{N_D = 0}$

$\boxed{V_D = 750 \text{ N}}$



$\Sigma M @ D = 0 \circlearrowleft +$

$50(6) - 800(2) - M_D = 0$

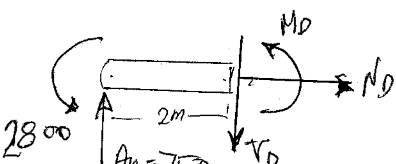
$\boxed{M_D = -1300 \text{ N-m}}$

$\boxed{N_E = 0}$

$\boxed{V_E = 50 \text{ N}}$

$\boxed{M_E = 100 \text{ N-m}}$

You could also solve for internal force at D using the LHS but you need to find reaction at support A.

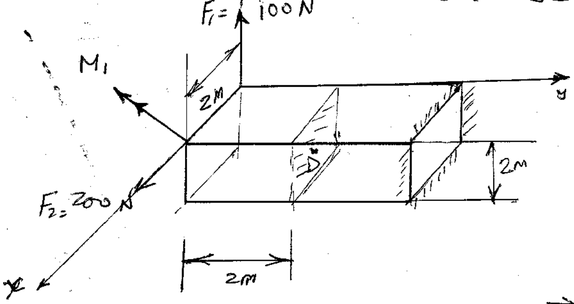


$\Sigma F_y = 0 \qquad V_D = 750$

$\Sigma F_x = 0 \qquad N_D = 0$

$\Sigma M @ D = 0 \circlearrowleft + \qquad M_D = -1300 \text{ N-m}$

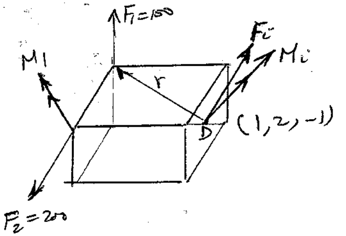
Internal forces in 3-D



$\vec{M}_1 = 20\vec{i} + 500\vec{j} - 100\vec{k}$

$\vec{F}_1 = 100\vec{k}$

$\vec{F}_2 = 200\vec{i}$



$\vec{F}_i$ = internal force

$\vec{M}_i$ = internal moment

$\vec{F}_i = F_{ix}\vec{i} + F_{iy}\vec{j} + F_{iz}\vec{k}$

$\vec{M}_i = M_{ix}\vec{i} + M_{iy}\vec{j} + M_{iz}\vec{k}$

$\sum \vec{F} = 0 \qquad \vec{F}_1 + \vec{F}_2 + \vec{F}_i = 0 \qquad r = -\vec{i} - 2\vec{j} + \vec{k}$

$\sum \vec{M}_{@D} = 0 \qquad \vec{r} \times \vec{F}_1 + r \times \vec{F}_2 + M_1 + M_i = 0$

$r \times (\vec{F}_1 + \vec{F}_2) + M_1 + M_i = 0$

$$\begin{vmatrix} i & j & k \\ -1 & -2 & 1 \\ 200 & 0 & 100 \end{vmatrix} + 20\vec{i} + 500\vec{j} - 100\vec{k} + M_x\vec{i} + M_y\vec{j} + M_z\vec{k} = 0$$

$-200\vec{i} + 300\vec{j} + 400\vec{k} + 20\vec{i} + 500\vec{j} - 100\vec{k} + M_x\vec{i} + M_y\vec{j} + M_z\vec{k} = 0$

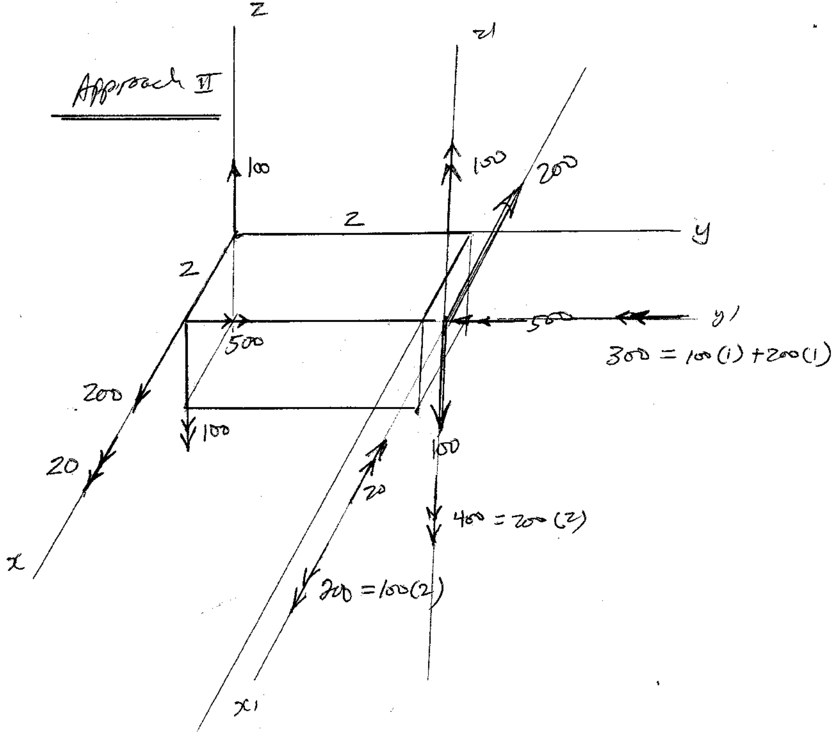
$$\boxed{(-200 + 20 + M_{ix})\vec{i} + (300 + 500 + M_y)\vec{j} + (400 - 100 + M_z)\vec{k} = 0}$$

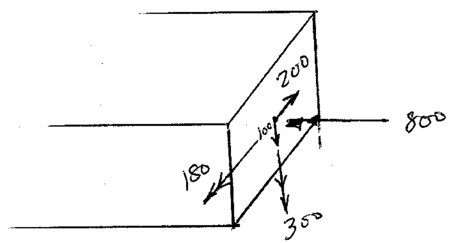
$200\vec{i} + 100\vec{k} + F_{ix}\vec{i} + F_{iy}\vec{j} + F_{iz}\vec{k} = 0$

$$\boxed{(200 + F_{ix})\vec{i} + (F_{iy})\vec{j} + (100 + F_{iz})\vec{k} = 0}$$

$$\boxed{F_{ix} = -200^{N},\quad F_{iy} = 0^{N},\quad F_{iz} = -100^{N},\quad M_{ix} = 180^{N\text{-}m},\quad M_{iy} = -800^{N\text{-}m},\quad M_{iz} = -300^{N\text{-}m}}$$

Approach II





$V_x = -200$	N	$M_x = 180$	N-m
$V_z = -100$	N	$M_y = -800$	N-m
$N_y = 0$	N	$M_z = -300$	N-m

Notes to remember when drawing SFD & BMD

(1) The value of shear at the starting end will be the value of support reaction (upward reaction is positive and downward is negative).

(2) The value of moment at the starting end will be the value of moment at the support (if hinge support moment will be zero) otherwise moment is positive if it develops tension in the bottom and compression at the top.

(3) Loading reduce shear by its magnitude (or its resultant) if applied downward and increase shear by its magnitude if applied upward.

(4) There is a jump in SFD at any concentrated force and a jump in BMD at any concentrated moment.

(5) Shear at B equals shear at A plus area of load between A & B and moment at B equals moment at A plus the area of shear between A & B.

(6) If no load, shear is flat and moment is linear
If load is uniform, shear is linear and moment is parabolic
If load is linear shear is 2nd degree & moment is third degree

(7) Slope of the shear equal to the load and slope of the moment is equal to the shear.

(8) $\dfrac{dV}{dx} = w(x) \quad , \quad \dfrac{dM}{\partial x} = V(x)$

$V_B = V_A$ + area of load between A & B.

$M_B = M_A$ + area of shear between A & B.

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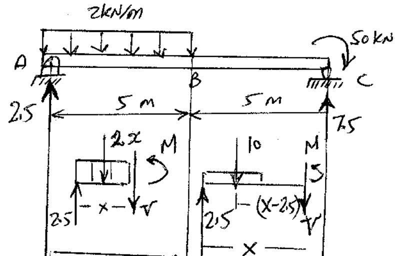


$\Sigma M @ C = 0$ $A_y = 2.5$ kN

$\Sigma F_y = 0$ $C_y = 7.5$ kN

$V = 0 \Rightarrow x = \frac{2.5}{2}$

$x = 1.25$

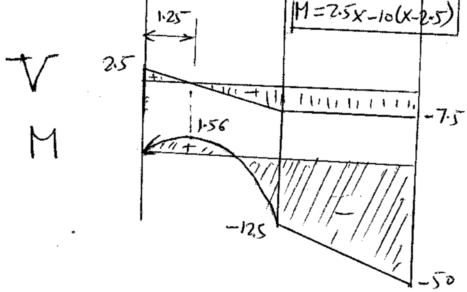
$\boxed{V = 2.5 - 2x}$

$\boxed{M = 2.5x - x^2}$

$V = 2.5 - 10$

$\boxed{V = -7.5}$

$\boxed{M = 2.5x - 10(x - 2.5)}$



SFD - Shear force diagram

BMD - Bending Moment diagram.

7.58
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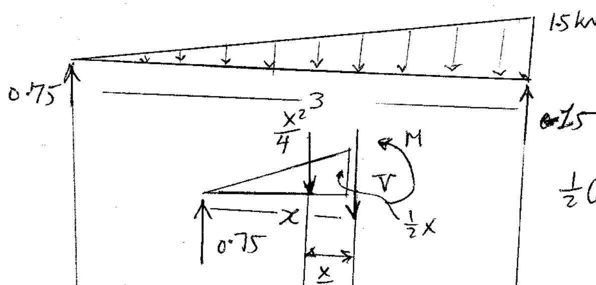
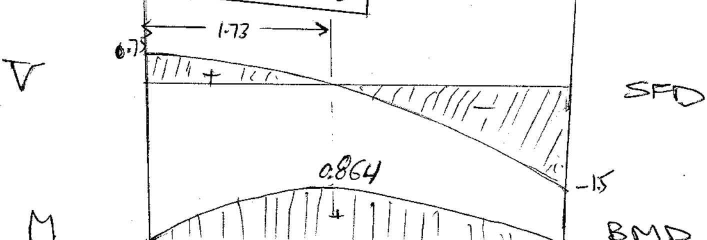


$\frac{1}{2}(x)\left(\frac{1}{2}x\right) = \frac{x^2}{4}$

$\boxed{V = 0.75 - \frac{x^2}{4}}$

$\boxed{M = 0.75x - \frac{x^3}{12}}$



SFD

BMD

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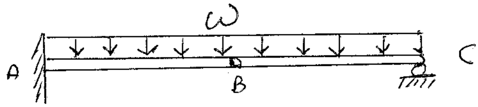
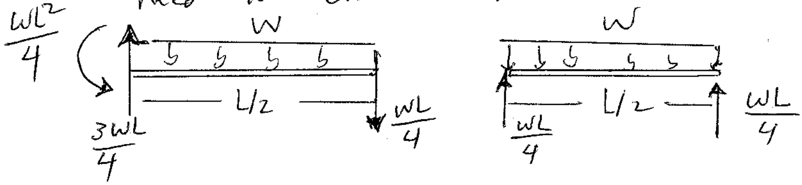
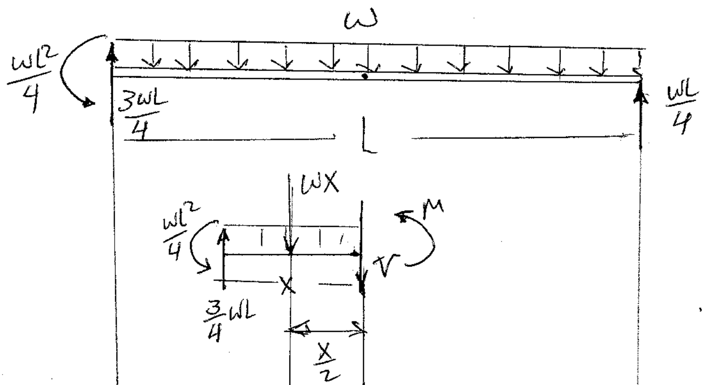


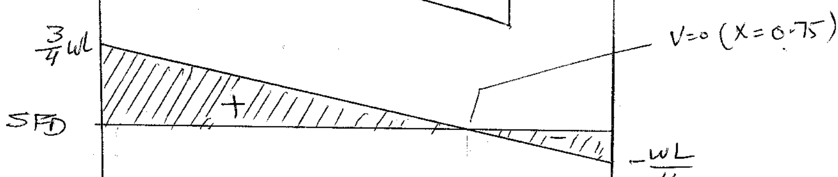
First find reactions at least at support A. need to disconnect members.

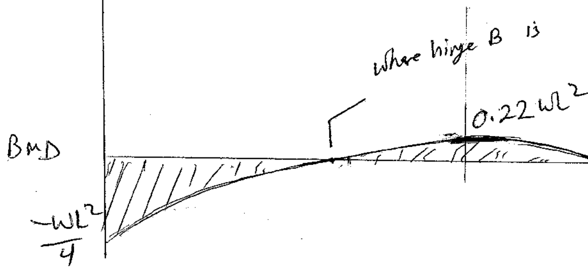




$$\boxed{V = \frac{3}{4}wL - wX}$$

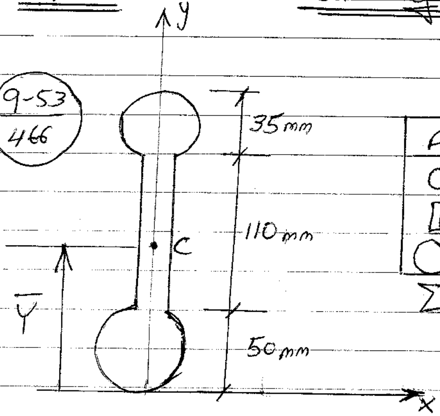
$$\boxed{M = \frac{-wL^2}{4} + \frac{3}{4}wLX - \frac{wX^2}{2}}$$





Ch. 9 : Center of Gravity & Centroid

9-53
466

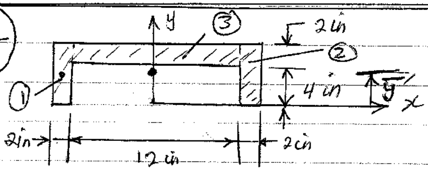


A_i	A_i	y_i	$A_i y_i$
○	962	177.5	170755
▯	1650	165	173250
◯	1963	25	49075
Σ	4575		393080

$$\bar{Y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{393080}{4575} = 85.9 \text{ mm}$$

$\bar{X} = 0$ (along y-axis)

9-63
468



$\bar{X} = 0$

$$\bar{Y}' = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{2(6)(3) + 2(6)(3) + 2(12)(5)}{12 + 12 + 24} = 4.0$$

$$\therefore \quad \bar{y} = 6 - \bar{y}' = 6 - 4 = 2.0 \text{ in}$$

or

$$\bar{Y}' = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = \frac{(16)(6)(3) - (12)(4)(2)}{(16)(6) - 12(4)} = 4.0$$

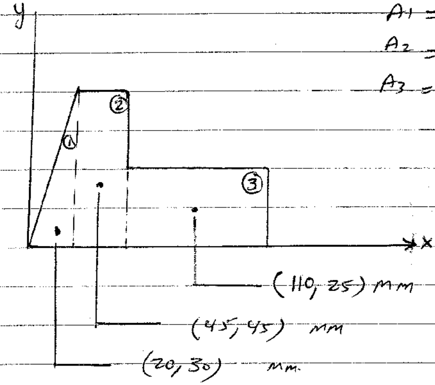
$$\therefore \quad \bar{y} = 6 - 4 = 2.0 \text{ in} \quad \text{(same results)}$$

7-65
469

$A_1 = \frac{1}{2}(30)(90) = 1350 \ mm^2$

$A_2 = 30(90) = 2700 \ mm^2$

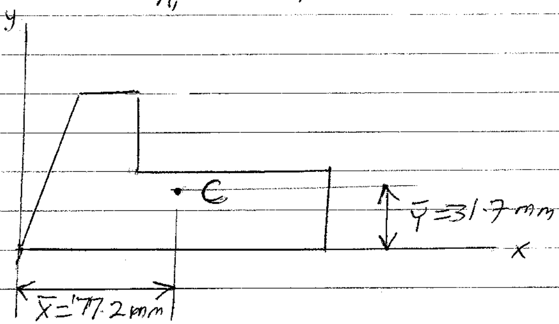
$A_3 = 50(100) = 5000 \ mm^2$



A_i	A_i	x_i	y_i	$A_i x_i$	$A_i y_i$
Δ	1350	20	30	27000	40500
▯	2700	45	45	121500	121500
▭	5000	110	25	550000	125000
Σ	9050			698500	287000

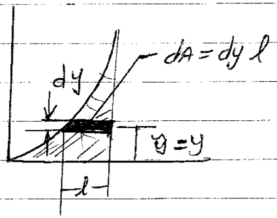
$$\bar{X} = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{698500}{9050} = 77.2 \ mm$$

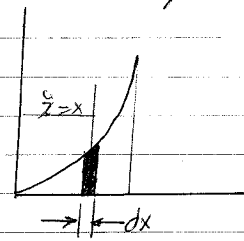
$$\bar{Y} = \frac{\Sigma A_i y_i}{A_i} = \frac{287000}{9050} = 31.7 \ mm$$



Finding the centroid using integration:

Remember that the element should be // to x-axis if $\bar{y}$ is needed and // to y-axis if $\bar{x}$ is needed. see figure below:





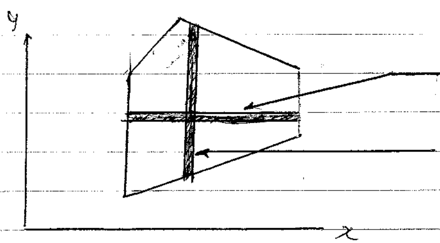
$$\bar{y} = \frac{\int \tilde{y}\, dA}{\int dA} = \frac{\int y\, l\, dy}{\int l\, dy}$$

y is constant for all points in the element.

$$\bar{x} = \frac{\int \tilde{x}\, dA}{\int dA} = \frac{\int x\, dA}{\int dA}$$

x is constant for all points in the element.

Ch.10 Moment of Inertia



$I_x = \int y^2 dA$, moment of inertia about x-axis

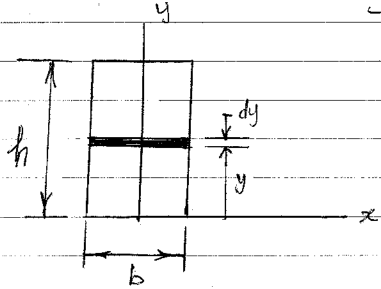
$I_y = \int x^2 dA$, moment of inertia about y-axis

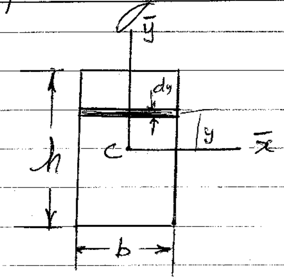
$I_z = I_x + I_y$, moment of inertia about z-axis (Polar moment of inertia)

I_x — choose element // to x-axis (constant y)

I_y — choose element // to y-axis (constant x).

Moment of inertia for rectangular section





$$I_x = \int y^2 dA$$

$$= \int_0^h y^2 b\, dy$$

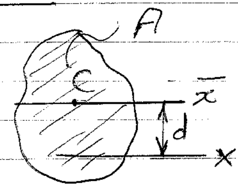
$$= \frac{bh^3}{3}$$

$$I_{\bar{x}} = \int y^2 dA$$

$$= \int_{-h/2}^{h/2} y^2 b\, dy = b \left| \frac{y^3}{3} \right|_{-h/2}^{h/2}$$

$$= \frac{bh^3}{12}$$

Parallel Axis Theorem



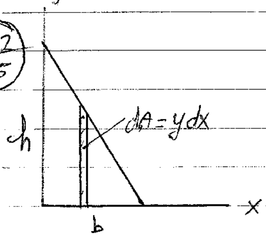
$$I_x = I_{\bar{x}} + Ad^2$$

$$= \frac{bh^3}{12} + bh\left(\frac{h}{2}\right)^2$$

$$= \frac{bh^3}{12} + \frac{bh^3}{4}$$

$$= \frac{bh^3}{3}$$ (same as above.

10-7
505



$$I_y = \int x^2 \, dA$$

$$= \int x^2 y \, dx$$

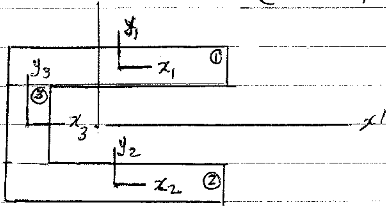
$$= \int_0^b x^2 \left(\frac{h}{b}\right)(b-x) \, dx$$

$$= \frac{h}{b} \int_0^b (x^2 b - x^3) \, dx$$

$$= \frac{h}{b} \left\{ \left| \frac{x^3 b}{3} \right|_0^b - \left| \frac{x^4}{4} \right|_0^b \right\}$$

$$= \frac{h}{b} \left\{ \frac{b^4}{3} - \frac{b^4}{4} \right\} = \frac{hb^3}{12}$$

-33
512



$$I_{x'} = (I_{x'})_1 + (I_{x'})_2 + (I_{x'})_3$$

$$= I_{x_1} + A_1 d_1^2 + I_{x_2} + A_2 d_2^2 + I_{x_3} + A_3 d_3^2$$

$$= 2\left(I_{x_1} + A_1 d_1^2\right) + I_{x_3} \qquad (d_3 = 0)$$

$$= 2\left(\frac{(40)^3(160)}{12} + (160)(40)(60)^2\right) + \frac{80^3 (40)}{12}$$

$$= 49.5 \times 10^6 \ \text{mm}^4$$