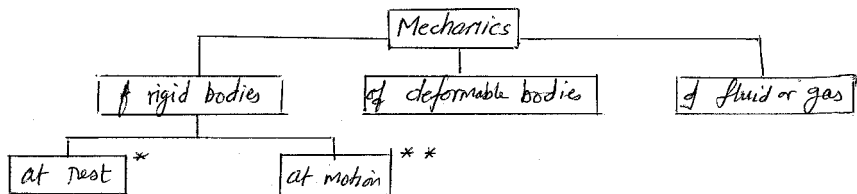


# Mechanics : The state of bodies at rest or motion



\* Mechanics of rigid bodies at rest = statics

\*\* Mechanics of rigid bodies at motion = dynamics

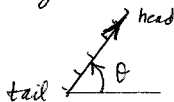
## 2.1 Scalars and Vectors

Physical quantities can be either scalar or vector quantities

Scalar : quantity which has only magnitude (mass, volume, time)

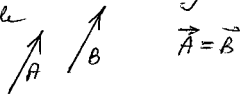
Vector : quantity which has both magnitude and direction (force, velocity)

A vector is represented graphically by an arrow whose length represent its magnitude and its angle with the +ve x-axis represents its direction



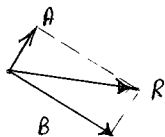
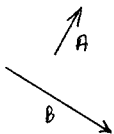
vector  $A$  is denoted by  $\vec{A}$  (where  $\rightarrow$  means  $A$  is a vector)  
magnitude of vector  $\vec{A}$  is denoted by  $|\vec{A}|$

① Two vectors  $\vec{A}$  and  $\vec{B}$  are equal when they have same magnitude and same angle



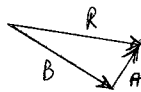
②  $\vec{A} = \alpha \vec{B}$  ( $\alpha$  = scalar quantity) vector  $\vec{A}$  is the same as vector  $B$  but  $|\vec{A}| = \alpha |\vec{B}|$

# Addition and Subtraction of vectors (Graphically)

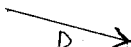


parallelogram

$$\vec{R} = \vec{A} + \vec{B}$$



triangle



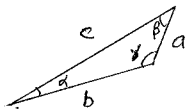
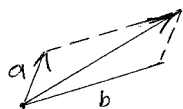
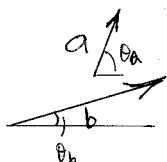
$$\vec{RF} = \vec{C} + \vec{D}$$

$\vec{C}$  and  $\vec{D}$  are called components and  $\vec{RF}$  is called resultant force.

two components may be combined together to give resultant  
resultant may be decompose to its component.

## Finding Resultant By Trigonometry

Find magnitude and direction of the resultant of  $\vec{a}$  and  $\vec{b}$   
(given both magnitude and direction of  $\vec{a}$  and  $\vec{b}$ )



six quantities ( $a, b, c, \alpha, \beta, \gamma$ ), one may find 2 of them if four of them provided through sine law and cosine law.

$$\gamma = 180^\circ - \theta_a + \theta_b$$

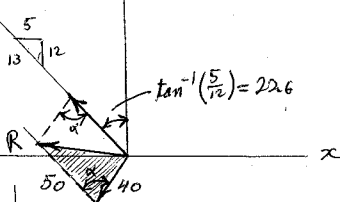
$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab \cos \gamma \\ \text{or } a^2 &= c^2 + b^2 - 2cb \cos \alpha \\ \text{or } b^2 &= c^2 + a^2 - 2ac \cos \beta \end{aligned}$$

cosine law.

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

sine law

$$\left( \frac{2-5}{25} \right)$$



$$\alpha = \frac{360 - 2(120)}{2} = 60$$

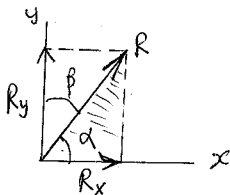
$$R = \sqrt{40^2 + 50^2 - 2(40)(50)\cos 60}$$

$$R = 45.82 \text{ lb} \quad \leftarrow$$

$$\frac{\sin 60}{45.82} = \frac{\sin \beta}{50} \Rightarrow \beta = 70.9$$

$$\theta = 120 - \beta + 22.6 + 90 \Rightarrow \theta = 161.2^\circ \quad \leftarrow$$

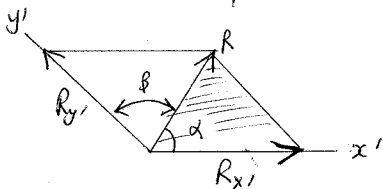
### Resolving Resultant to components



$R_x$  and  $R_y$  component of  $R$  along  $x$  and  $y$  axis respectively

$$\frac{R}{\sin 90} = \frac{R_y}{\sin \alpha} = \frac{R_y}{\cos \beta} \Rightarrow R_y = R \cos \beta$$

$$\frac{R}{\sin 90} = \frac{R_x}{\sin \beta} = \frac{R_x}{\cos \alpha} \Rightarrow R_x = R \cos \alpha$$

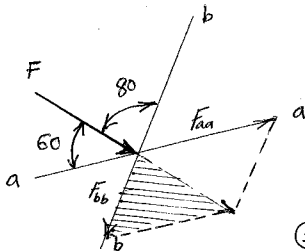


$$R_{x'} \neq R \cos \alpha$$

$$R_{y'} \neq R \cos \beta$$

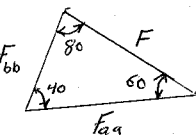
$$\frac{R}{\sin [180 - (\alpha + \beta)]} = \frac{R_{x'}}{\sin \beta} = \frac{R_{y'}}{\sin \alpha}$$

$\frac{2-18}{27}$



Given  $F_a = 30$   
Find  $F_b = ?$  and  $F = ?$

- ①  $F$  should be the diagonal of the parallelogram
- ②  $F_a$  and  $F_b$  should form sides of the parallelogram
- ③  $F, F_a, F_b$  joined by their tails



$$\frac{F_a}{\sin 80} = \frac{F}{\sin 40}$$

$$\frac{30}{\sin 80} = \frac{F}{\sin 40}$$

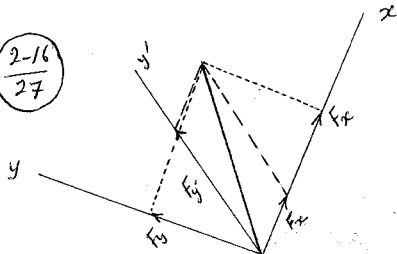
$$\Rightarrow F = 19.58 \text{ lb}$$

$$\frac{30}{\sin 80} = \frac{F_b}{\sin 60}$$

$\Rightarrow$

$$F_b = 26.4 \text{ lb}$$

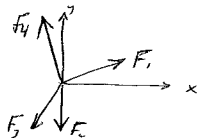
$\frac{2-16}{27}$



## 2.4 Addition of a System of Coplanar Forces

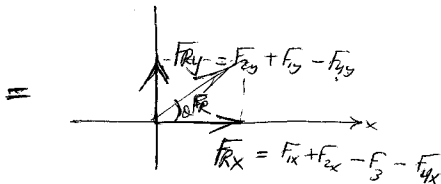
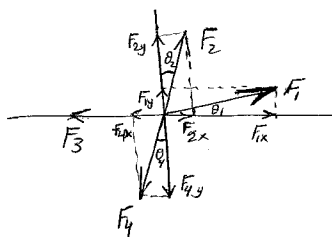
In section 2.3 we have learned about the method of parallelogram in finding resultant at resolution of a force to its components along specified axis. This method seems to be complicated when resultant of more than two forces is required. Therefore we need another technique to find resultant which will be described below.

Coplanar Forces: Are forces that lie in the same plane



$F_1, F_2, F_3$  and  $F_4$  are example of coplanar forces.

Scalar Approach



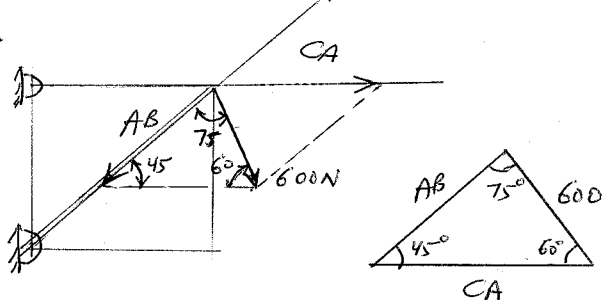
$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

$$\theta = \tan^{-1} \left( \frac{F_{Ry}}{F_{Rx}} \right)$$

Vector Approach

$$\begin{aligned} \vec{F}_R &= F_{Rx} \vec{i} + F_{Ry} \vec{j} \\ \vec{F}_R &= \sum F_{ix} \vec{i} + \sum F_{iy} \vec{j} \end{aligned} \quad \left\{ \begin{aligned} |\vec{F}_R| &= \sqrt{F_{Rx}^2 + F_{Ry}^2} \\ \theta &= \tan^{-1} \left( \frac{F_{Ry}}{F_{Rx}} \right) \end{aligned} \right.$$

$$\frac{2-14}{27}$$

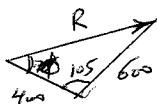
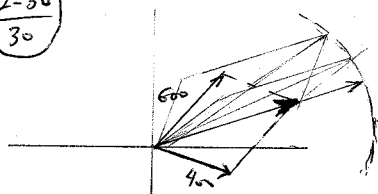


Using Sine law twice:

$$\frac{600}{\sin 45^\circ} = \frac{AB}{\sin 60^\circ} \Rightarrow AB = 735 \text{ N}$$

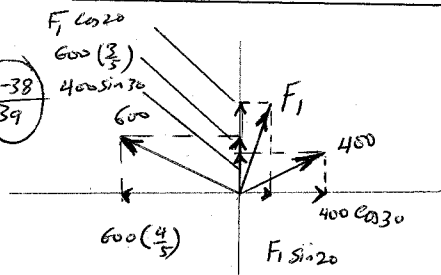
$$\frac{600}{\sin 45^\circ} = \frac{CA}{\sin 75^\circ} \Rightarrow CA = 820 \text{ N}$$

$$\frac{2-30}{30}$$

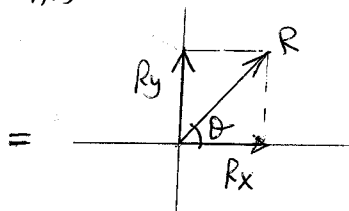


$$R + F = 900$$

$$\frac{2-38}{39}$$



$$F = 500$$



[2]

[1]

$$R_x = 400 \cos 30^\circ + 500 (\sin 20^\circ) - 600 \left(\frac{4}{5}\right) = 37.4$$

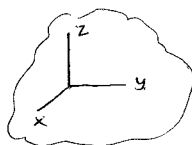
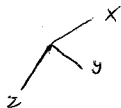
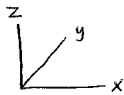
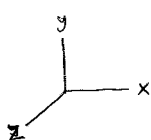
$$R_y = 500 \cos 20^\circ + 600 \left(\frac{3}{5}\right) + 400 \sin 30^\circ = 1029.8$$

$$R = \sqrt{37.4^2 + 1029.8^2} = 1031 \text{ N}$$

$$\theta = \tan^{-1} \left( \frac{1029.8}{37.4} \right) = 87.9^\circ$$

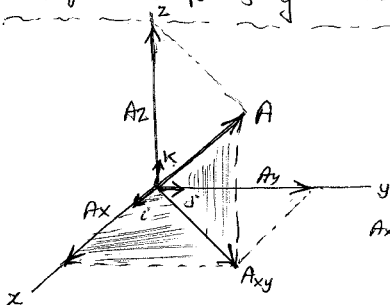
## 2.5 Cartesian Vectors - 3D

We are going to use the right-handed cartesian coordinate system where when the right hand fingers are curled from  $x$  to  $y$  your thumb will point toward the  $z$ -direction.



can be used in this textbook.

### Rectangular components of a vector in 3-D



Let  $\vec{i}, \vec{j}, \vec{k}$  be unit vectors along  $x, y, z$  axes respectively.

$A_x, A_y, A_z$  are components of vector  $A$

$$\vec{A} = (A_x, A_y, A_z)$$

where

$A_x$  = component of vector  $A$  along +ve  $x$ -axis

$A_y$  = " " " " " "  $y$ -axis

$A_z$  = " " " " " "  $z$ -axis

$$\boxed{1} \quad \vec{A} = \underbrace{A_x \vec{i} + A_y \vec{j}}_{\vec{A}_{xy}, \text{ vector in the } x-y \text{ plane}} + A_z \vec{k} \quad \text{--- (1)}, \text{ one may think that vector } A \text{ is the sum of three vectors}$$

unit vector: Any vector which has magnitude of unity (=1)

example  $\vec{i}, \vec{j}, \vec{k}$  each has a magnitude of one

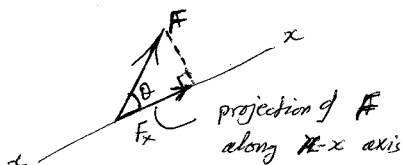
$$|\vec{i}| = |\vec{j}| = |\vec{k}| = 1$$

\* Any vector can be made unit vector if one divides itself by its magn.

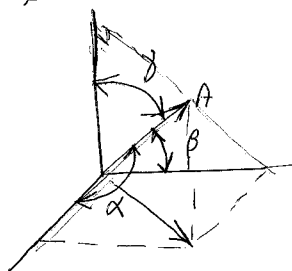
$$\vec{F} \Rightarrow \vec{u}_F = \vec{F} / |\vec{F}|, \text{ unit vector of } \vec{F}$$

$$\vec{B} \Rightarrow \vec{u}_B = \vec{B} / |\vec{B}|, \text{ unit vector of } \vec{B}$$

magnitude of  $\vec{A} = |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$



$$F_x = |F| \cos \beta$$



let  $\alpha$  = angle between  $A$  and +ve  $x$ -axis,  
 $\beta$  = angle between  $A$  and +ve  $y$ -axis  
 $\gamma$  = angle between  $A$  and +ve  $z$ -axis

$$\left. \begin{aligned} \text{projection of } A \text{ along } x\text{-axis} &= A_x = |A| \cos \alpha \\ \text{projection of } A \text{ along } y\text{-axis} &= A_y = |A| \cos \beta \\ \text{projection of } A \text{ along } z\text{-axis} &= A_z = |A| \cos \gamma \end{aligned} \right\} \quad \text{--- (2)}$$

See Figure 2-28 page 43 (Remember here projection like components)

Substitution of (2) into (1) yield .

$$\begin{aligned} \vec{A} &= |A| \cos \alpha \vec{i} + |A| \cos \beta \vec{j} + |A| \cos \gamma \vec{k} \\ &= |A| \{ \cos \alpha \vec{i} + \cos \beta \vec{j} + \cos \gamma \vec{k} \} \end{aligned}$$

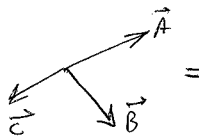
$$\frac{\vec{A}}{|A|} = \{ \cos \alpha \vec{i} + \cos \beta \vec{j} + \cos \gamma \vec{k} \} = \vec{U}_A$$

$$\Rightarrow \boxed{2} \quad \vec{A} = |A| \vec{U}_A, \quad \vec{U}_A = \cos \alpha \vec{i} + \cos \beta \vec{j} + \cos \gamma \vec{k}$$

$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$ ,  
 one needs only to define  
 two angles out of three  
 to determine direction of  
 a vector



## 2.6 Addition and Subtraction of Cartesian Vectors



$$\vec{D} = \vec{A} + \vec{B} + \vec{C}$$

$$\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$$

$$\vec{B} = B_x \vec{i} + B_y \vec{j} + B_z \vec{k}$$

$$\vec{C} = C_x \vec{i} + C_y \vec{j} + C_z \vec{k}$$

$$\vec{D} = (A_x + B_x + C_x) \vec{i} + (A_y + B_y + C_y) \vec{j} + (A_z + B_z + C_z) \vec{k}$$

Example; ① Find the resultant of two forces in vector form

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2, \text{ given } \vec{F}_1 = 5\vec{i} + 4\vec{j} - 2\vec{k}$$

$$\vec{F}_2 = 3\vec{i} + 4\vec{j} + 2\vec{k}$$

$$\vec{F}_R = 8\vec{i} + 7\vec{j}$$

$$\text{let } \vec{C} = \vec{F}_1 - \vec{F}_2 = 2\vec{i} - 4\vec{k}$$

$$(\alpha, \beta, \gamma)$$

② Find the magnitude and direction of the above resultant

$$\text{given } \vec{F}_R = 8\vec{i} + 7\vec{j}$$

$$|\vec{F}_R| = \sqrt{8^2 + 7^2}$$

$$\vec{u}_R = \vec{F}_R / |\vec{F}_R| = \frac{8\vec{i} + 7\vec{j}}{\sqrt{113}} = \frac{8}{\sqrt{113}} \vec{i} + \frac{7}{\sqrt{113}} \vec{j}$$

$$\vec{u} = \cos \alpha \vec{i} + \cos \beta \vec{j}$$

∴ direction

$$\cos \alpha = \frac{8}{\sqrt{113}} \Rightarrow$$

$$\alpha = \cos^{-1} \left( \frac{0.7526}{0.7882} \right) = 41.2^\circ$$

$$\cos \beta = \frac{7}{\sqrt{113}} \Rightarrow$$

$$\beta = \cos^{-1} \left( \frac{0.6585}{0.6897} \right) = 48.8^\circ$$

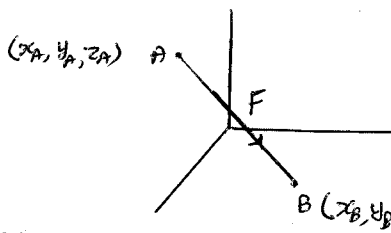
$$\gamma = 90^\circ$$

## 2.7 Position Vectors $\vec{r}$

In previous lecture we talked about a vector or a force which can be written in general as:

$$\vec{F} = |\vec{F}| \vec{u}, \quad \vec{u} = \cos \alpha \vec{i} + \cos \beta \vec{j} + \cos \gamma \vec{k}$$

What about if the angles were not provided explicitly, and instead we are given the coordinates of two points along which the force is acting.



Steps

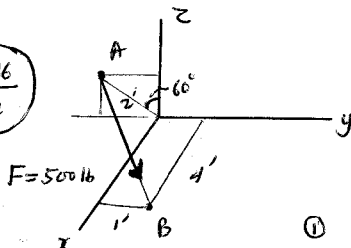
$$(1) \vec{r}_{AB} = (x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} + (z_B - z_A)\vec{k}$$

$$(2) |\vec{r}_{AB}| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$$

$$(3) \vec{u}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \frac{(1)}{(2)}$$

$$(4) \vec{F}_{AB} = \vec{u}_{AB} |\vec{F}|$$

$$\frac{2-86}{62}$$



$$A = (0, 1, 1)$$

$$B = (4, 1, 0)$$

Express  $F$  in vector form & determine its coordinate direction angles.

$$(1) \vec{r}_{AB} = 4\vec{i} + 2\vec{j} - \vec{k}$$

$$(2) |\vec{r}_{AB}| = \sqrt{4^2 + 2^2 + (-1)^2} = 4.94$$

$$(3) \vec{u} = \vec{r}/|\vec{r}| = 0.809\vec{i} + 0.553\vec{j} - 0.202\vec{k}$$

$$(4) \vec{F} = 500\vec{u} = 404\vec{i} + 276\vec{j} - 101\vec{k}$$

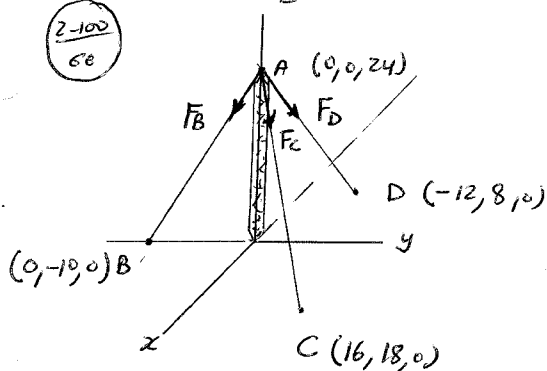
$$\text{But } \vec{u} = \cos \alpha \vec{i} + \cos \beta \vec{j} + \cos \gamma \vec{k}$$

$$\alpha = \cos^{-1}(0.809) = 36^\circ$$

$$\beta = \cos^{-1}(0.553) = 56.5^\circ$$

$$\gamma = \cos^{-1}(-0.202) = 101.6^\circ$$

$$\frac{2-100}{68}$$



$$\vec{F}_B = |\vec{F}_B| \vec{u}_{AB} = 520 \left\{ \frac{0\vec{i} - 10\vec{j} - 24\vec{k}}{26} \right\} = 0\vec{i} - 200\vec{j} - 480\vec{k}$$

$$\vec{F}_C = |\vec{F}_C| \vec{u}_{AC} = 680 \left\{ \frac{16\vec{i} + 18\vec{j} - 24\vec{k}}{34} \right\} = 320\vec{i} + 360\vec{j} - 480\vec{k}$$

$$\vec{F}_D = |\vec{F}_D| \vec{u}_{AD} = 560 \left\{ \frac{-12\vec{i} + 8\vec{j} - 24\vec{k}}{28} \right\} = -240\vec{i} + 160\vec{j} - 480\vec{k}$$

$$\vec{R} = \vec{F}_B + \vec{F}_C + \vec{F}_D = 80\vec{i} + 320\vec{j} - 1440\vec{k}$$

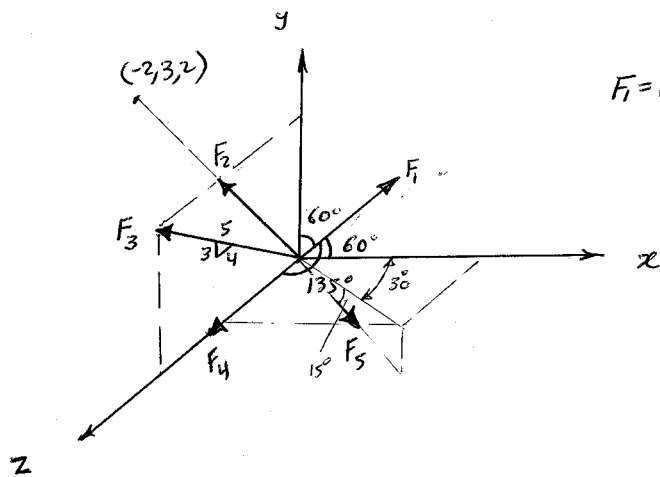
$$|\vec{R}| = 1477.3$$

← Magnitude

$$\vec{u}_R = 0.0541\vec{i} + 0.2166\vec{j} - 0.975\vec{k}$$

$$\left. \begin{aligned} \alpha &= \cos^{-1}(0.0541) = 86.9^\circ \\ \beta &= \cos^{-1}(0.2166) = 77.5^\circ \\ \gamma &= \cos^{-1}(0.975) = 12.84^\circ \end{aligned} \right\} \leftarrow \text{Directions}$$

# Resultant of a system of forces (3-dimensions)



$$F_1 = F_2 = F_3 = F_4 = F_5 = 100$$

One needs to express each force in cartesian vector form:

$$\vec{F} = |\vec{F}| \vec{u}_F \quad \left[ \begin{array}{l} \vec{u}_F = \cos \alpha \vec{i} + \cos \beta \vec{j} + \cos \gamma \vec{k} \\ \vec{u}_F = \vec{r} / |\vec{r}| \end{array} \right.$$

$$\vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$$

$$\vec{F}_1 = 100 \vec{u}_{F_1} = 100 (\cos 60^\circ \vec{i} + \cos 60^\circ \vec{j} + \cos 135^\circ \vec{k})$$

$$\vec{F}_2 = 100 \vec{u}_{F_2} = 100 \left( \frac{-2\vec{i} + 3\vec{j} + 2\vec{k}}{\sqrt{4+9+4}} \right) = 100 (-.485\vec{i} + .728\vec{j} + .485\vec{k})$$

$$\vec{F}_3 = \frac{3}{5} (100) \vec{j} + \frac{4}{5} (100) \vec{k}$$

$$\vec{F}_4 = 100 \vec{k}$$

$$\vec{F}_5 = (100 \cos 15^\circ) \cos 30^\circ \vec{i} - 100 \sin 15^\circ \vec{j} + (100 \cos 15^\circ) \cos 60^\circ \vec{k}$$

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5$$

$$= 100 (\cos 60 - .485 + 0 + 0 + \cos 15 \cos 30) \vec{i} \\ + 100 (\cos 60 + .728 + \frac{3}{5} + 0 + \sin 15) \vec{j} \\ + 100 (\cos 135 + .485 + \frac{4}{5} + 1 + \cos 15 \cos 60) \vec{k}$$

$$\vec{R} = 85.15 \vec{i} + 208.7 \vec{j} + 206.1 \vec{k}$$

$$|\vec{R}| = \sqrt{(85.15)^2 + (208.7)^2 + (206.1)^2} = 305.4$$

$$\vec{u}_R = \frac{\vec{R}}{|\vec{R}|} = .279 \vec{i} + .683 \vec{j} + .675 \vec{k}$$

$$\vec{u}_R = \cos \alpha \vec{i} + \cos \beta \vec{j} + \cos \gamma \vec{k}$$

$$\alpha = \cos^{-1}(.279) = 73.8^\circ$$

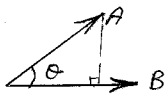
$$\beta = \cos^{-1}(.683) = 46.9^\circ$$

$$\gamma = \cos^{-1}(.675) = 47.55^\circ$$

$$\vec{R} = R_x \vec{i} + R_y \vec{j} + R_z \vec{k}$$

$$\vec{R} = \sum F_x \vec{i} + \sum F_y \vec{j} + \sum F_z \vec{k}$$

## 2.9 Dot Product



$$\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$$

$$\vec{B} = B_x \vec{i} + B_y \vec{j} + B_z \vec{k}$$

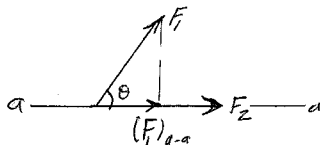
$$\left. \begin{array}{l} \vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k} \\ \vec{B} = B_x \vec{i} + B_y \vec{j} + B_z \vec{k} \end{array} \right\} \vec{A} \cdot \vec{B} = (A_x B_x + A_y B_y + A_z B_z) \quad (1)$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta \Rightarrow$$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

$$\theta = \cos^{-1} \left( \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} \right)$$

$$\theta = \cos^{-1} (\vec{u}_A \cdot \vec{u}_B)$$

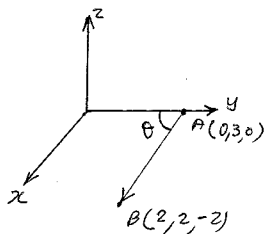


$(F_i)_{a-a}$  = the projection of  $F_i$  along  $a-a$

$$(F_i)_{a-a} = F_i \cos \theta$$

$$(F_i)_{a-a} = \vec{F}_i \cdot \vec{u}_{a-a} \quad (= F_i \cos \theta)$$

$$(\vec{F})_{a-a} = [\vec{F} \cdot \vec{u}_{a-a}] \vec{u}_{a-a}$$



Find  $\theta = ?$

$$\vec{r}_{AB} = 2\vec{i} - \vec{j} - 2\vec{k}, \quad |\vec{r}_{AB}| = 3$$

$$\vec{u}_{AB} = \frac{2}{3}\vec{i} - \frac{1}{3}\vec{j} - \frac{2}{3}\vec{k}$$

$$\vec{u}_y = -\vec{j}$$

$$\cos \theta = \vec{u}_{AB} \cdot \vec{u}_y$$

$$\theta = \cos^{-1} (\vec{u}_{AB} \cdot \vec{u}_y) = \cos^{-1} \left( -\frac{1}{3} \right) = 70.5^\circ$$

The projection of AB along  $y = (\vec{AB} \cdot \vec{u}_y) = -1$

The projection of AB along  $y$  axis in vector form  $= -\vec{j}$

2-113  
T3

## Ch.3 Equilibrium of a Particle

3.1

In this chapter we are going to apply what we have learned in chapter (2) to solve some real problems.

To maintain equilibrium in general, it is necessary to satisfy Newton's first law which states that if the resultant force acting on a body (or particle) is zero, then the body (or particle) is in equilibrium.

$$\vec{F}_R = \vec{0}$$

or

$$\sum \vec{F} = \vec{0}$$

A vector has a magnitude zero if its individual components are equal to zeros.

$$\vec{F}_R = F_{Rx} \vec{i} + F_{Ry} \vec{j} + F_{Rz} \vec{k} = 0\vec{i} + 0\vec{j} + 0\vec{k}$$

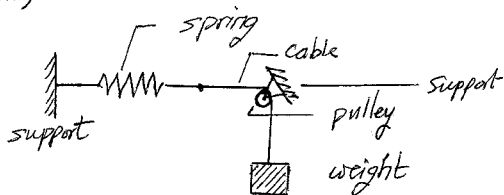
$$\therefore F_{Rx} = 0 \quad (\text{or } \sum F_x = 0)$$

$$F_{Ry} = 0 \quad (\text{or } \sum F_y = 0)$$

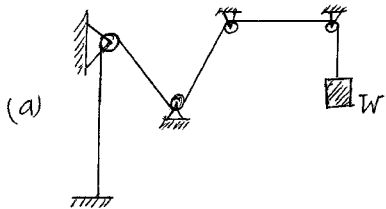
$$F_{Rz} = 0 \quad (\text{or } \sum F_z = 0)$$

### 3.2 The Free-Body Diagram (F.B.D)

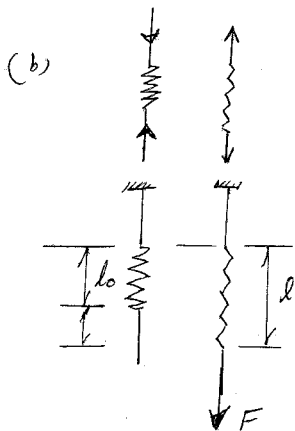
FBD is a sketch of the particle after isolation from its surrounding showing all the acting forces (which are known) and the reactive forces (which are usually unknown).



# Forces on Cables (Cords, ropes) & Spring.



cables can support only tension force and this force act in the direction of the cable and constant throughout its entire length.



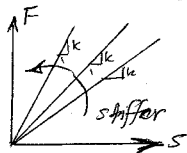
spring can take tension or compression and its change in length is proportional to the force

$$\text{change in length} = l - l_0 = s$$

$$F \propto s$$

$$F = ks = k(l - l_0)$$

$k$  = stiffness of spring.



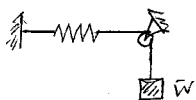
$$l - l_0 > 0 \quad (\text{tension})$$

$$l - l_0 < 0 \quad (\text{compression})$$

## Steps for Drawing F.B.D.

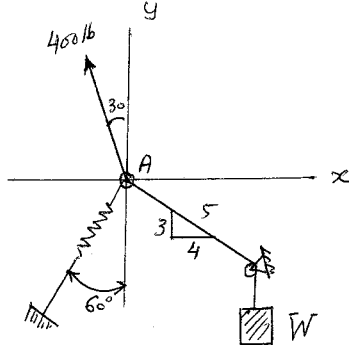
- ① Draw the general shape of the particle after isolation
- ② indicate on the sketch all the active and reactive forces
- ③ Apply equations of equilibrium.

Ex.



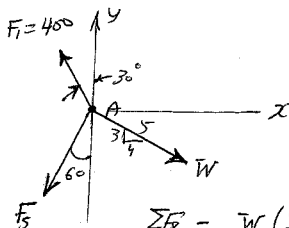
<p>F.B.D of spring</p>	<p>F.B.D of pulley</p>	<p>F.B.D of W</p>
------------------------	------------------------	-------------------





Particle A is subjected to three forces ( $W$ ,  $F_{\text{spring}}$ ,  $400 \text{ lb}$ )

Determine the necessary weight  $W$  and the force in the spring to maintain the particle in equilibrium.



For particle A to be in equilibrium

$$\left. \begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \end{aligned} \right\} \text{Approach (I)}$$

$$\sum F_x = W\left(\frac{4}{5}\right) - F_s \sin 60 - 400 \sin 30 = 0 \quad (1)$$

$$\sum F_y = -W\left(\frac{3}{5}\right) - F_s \cos 60 + 400 \cos 30 = 0 \quad (2)$$

Solving (1) & (2)  $\Rightarrow W = 435 \text{ lb} \quad \& \quad F_s = 171 \text{ lb}$

Approach (II) : Resultant.

$$\vec{F}_1 = -400 \sin 30 \vec{i} + 400 \cos 30 \vec{j}$$

$$\vec{W} = \frac{4}{5}W \vec{i} - \frac{3}{5}W \vec{j}$$

$$\vec{F}_s = -F_s \sin 60 \vec{i} - F_s \cos 60 \vec{j}$$

$$\vec{R} = \vec{F}_1 + \vec{W} + \vec{F}_s = \left(\frac{4}{5}W - F_s \sin 60 - 400 \sin 30\right) \vec{i} + \left(-\frac{3}{5}W - F_s \cos 60 + 400 \cos 30\right) \vec{j}$$

For Equilibrium:  $\vec{R} = 0 \quad (R_x = 0, R_y = 0)$

$$\therefore \frac{4}{5}W - F_s \sin 60 - 400 \sin 30 = 0 \quad (R_x = 0) \quad (3)$$

$$-\frac{3}{5}W - F_s \cos 60 + 400 \cos 30 = 0 \quad (R_y = 0) \quad (4)$$

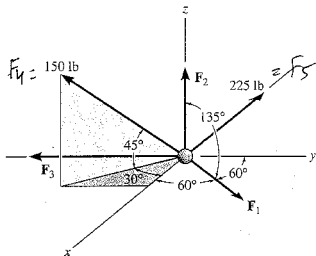
$\therefore$  (1) & (2) are same as 3 & 4

$\therefore \boxed{W = 435 \text{ lb}} \quad \& \quad \boxed{F_s = 171 \text{ lb}}$

Approach I : Scalar approach :

Approach II : Vector approach :

3-42. Determine the magnitudes of  $F_1$ ,  $F_2$ , and  $F_3$  for equilibrium of the particle.



$$\vec{F}_1 = |\vec{F}_1| [\cos 60^\circ \vec{i} + \cos 60^\circ \vec{j} + \cos 135^\circ \vec{k}]$$

$$\vec{F}_2 = |\vec{F}_2| \vec{k}$$

$$\vec{F}_3 = -|\vec{F}_3| \vec{i}$$

$$\vec{F}_4 = (150 \cos 45^\circ) \cos 30^\circ \vec{i} - (150 \cos 45^\circ) \sin 30^\circ \vec{j} + 150 \sin 45^\circ \vec{k}$$

$$\vec{F}_5 = -225 \vec{i}$$

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5$$

$$\begin{aligned} \vec{R} &= (|\vec{F}_1| \cos 60^\circ + 0 + 0 + (150 \cos 45^\circ) \cos 30^\circ - 225) \vec{i} \\ &\quad (|\vec{F}_1| \cos 60^\circ + 0 - |\vec{F}_3| - (150 \cos 45^\circ) \sin 30^\circ) \vec{j} + \\ &\quad (|\vec{F}_1| \cos 135^\circ + |\vec{F}_2| + 0 + 150 \sin 45^\circ) \vec{k} \end{aligned}$$

For equilibrium  $\vec{R} = \vec{0} = 0\vec{i} + 0\vec{j} + 0\vec{k}$

$$\sum F_x = 0 \quad |\vec{F}_1| \cos 60^\circ + (150 \cos 45^\circ) \cos 30^\circ - 225 \Rightarrow |\vec{F}_1| = 266 \text{ lb}$$

$$\sum F_y = 0 \quad |\vec{F}_1| \cos 60^\circ - |\vec{F}_3| - (150 \cos 45^\circ) \sin 30^\circ = 0 \Rightarrow |\vec{F}_3| = 80.1 \text{ lb}$$

$$\sum F_z = 0 \quad |\vec{F}_1| \cos 135^\circ + |\vec{F}_2| + 150 \sin 45^\circ = 0 \Rightarrow |\vec{F}_2| = 82.2 \text{ lb}$$

3-49. Determine the force in each cable needed to support the 500-lb cylinder.

3-49  
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$$A = (0, 0, 0)$$

$$B = (-3, 6, 6)$$

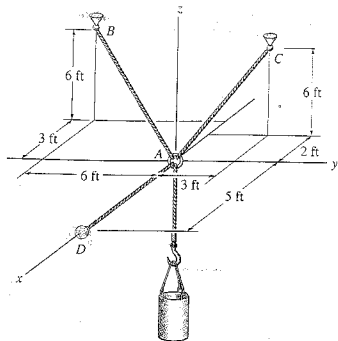
$$C = (-2, 3, 6)$$

$$D = (5, 0, 0)$$

$$\vec{u}_{AB} = -0.333\vec{i} - 0.666\vec{j} + 0.666\vec{k}$$

$$\vec{u}_{AC} = -0.2857\vec{i} + 0.4286\vec{j} + 0.857\vec{k}$$

$$\vec{u}_{AD} = \vec{i}$$



Prob. 3-49

$$\vec{F}_{AB} = -0.333 |\vec{F}_{AB}| \vec{i} - 0.666 |\vec{F}_{AB}| \vec{j} + 0.666 |\vec{F}_{AB}| \vec{k}$$

$$\vec{F}_{AC} = -0.2857 |\vec{F}_{AC}| \vec{i} + 0.4286 |\vec{F}_{AC}| \vec{j} + 0.857 |\vec{F}_{AC}| \vec{k}$$

$$\vec{F}_{AD} = |\vec{F}_{AD}| \vec{i}$$

$$\vec{W} = -500 \vec{k}$$

$$\vec{R} = \vec{F}_{AB} + \vec{F}_{AC} + \vec{F}_{AD} + \vec{W}$$

$$\vec{R} = [-0.333 |\vec{F}_{AB}| - 0.2857 |\vec{F}_{AC}| + |\vec{F}_{AD}|] \vec{i} + [-0.666 |\vec{F}_{AB}| + 0.4286 |\vec{F}_{AC}|] \vec{j} + [0.666 |\vec{F}_{AB}| + 0.857 |\vec{F}_{AC}| - 500] \vec{k}$$

For equilibrium  $\vec{R} = 0\vec{i} + 0\vec{j} + 0\vec{k}$

$$-0.333 |\vec{F}_{AB}| - 0.2857 |\vec{F}_{AC}| + |\vec{F}_{AD}| = 0 \quad (1)$$

$$-0.666 |\vec{F}_{AB}| + 0.4286 |\vec{F}_{AC}| = 0 \quad (2)$$

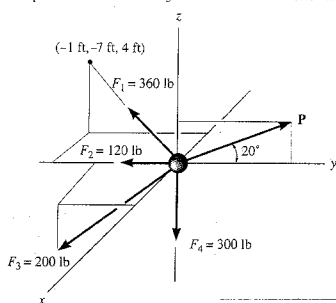
$$+0.666 |\vec{F}_{AB}| + 0.857 |\vec{F}_{AC}| - 500 = 0 \quad (3)$$

$$(2) + (3) \Rightarrow |\vec{F}_{AC}| = 389 \text{ lb}$$

$$\text{Substitute into (2) or (3)} \Rightarrow |\vec{F}_{AB}| = 250 \text{ lb}$$

$$\text{Substitute into (1)} \Rightarrow \vec{F}_{AD} = 194 \text{ lb}$$

3-73. Determine the magnitude of  $P$  and the coordinate direction angles of the 200-lb force required for equilibrium of the particle. Note that  $F_3$  acts in the octant shown.



$$\vec{F}_1 = 360 \left\{ \frac{1}{\sqrt{66}} \vec{i} - \frac{7}{\sqrt{66}} \vec{j} + \frac{4}{\sqrt{66}} \vec{k} \right\}$$

$$\vec{F}_1 = -44.3 \vec{i} - 310 \vec{j} + 177 \vec{k} \quad (a)$$

$$\vec{F}_2 = +0 \vec{i} - 120 \vec{j} + 0 \vec{k} \quad (b)$$

$$\vec{F}_3 = F_{3x} \vec{i} - F_{3y} \vec{j} - F_{3z} \vec{k} \quad (c)$$

$$\vec{P} = 0 \vec{i} + P \cos 20^\circ \vec{j} + P \sin 20^\circ \vec{k} \quad (d)$$

$$\vec{F}_4 = -300 \vec{k}$$

$$\sum F_x = 0: -44.3 + F_{3x} = 0 \quad (1)$$

$$\sum F_y = 0: -310 - 120 - F_{3y} + P \cos 20^\circ = 0 \quad (2)$$

$$\sum F_z = 0: 177 - F_{3z} + P \sin 20^\circ - 300 = 0 \quad (3)$$

$$\text{from (1)} \quad F_{3x} = 44.3$$

$$\text{from (2)} \quad F_{3y} = P \cos 20^\circ - 430$$

$$\text{from (3)} \quad F_{3z} = P \sin 20^\circ - 123$$

$$\text{But } |\vec{F}_3|^2 = F_{3x}^2 + F_{3y}^2 + F_{3z}^2$$

$$(200)^2 = (44.3)^2 + [P \cos 20^\circ - 430]^2 + [P \sin 20^\circ - 123]^2$$

$$\text{Solve for } P = 639 \text{ lb}$$

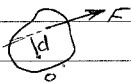
$$\text{Substituting in (2) \& (3)} \Rightarrow F_{3y} = 120.5 \quad F_{3z} = 95.5 \text{ lb}$$

now can you find  $\alpha, \beta, \gamma$  for  $\vec{F}_3$ ?

## Ch. 4 Force System Resultants

### ① Moment of a force - scalar formulation

$$M_o = \underbrace{Fd}_{\text{magnitude}} \underbrace{\curvearrowright}_{\text{direction}}$$



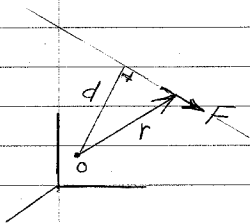
where  $d$  is the shortest distance between  $F$  and the point.

### ② Moment of a force - vector formulation

$$\vec{M}_o = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$\begin{aligned} \vec{r} &= r_x \hat{i} + r_y \hat{j} + r_z \hat{k} \\ \vec{F} &= F_x \hat{i} + F_y \hat{j} + F_z \hat{k} \end{aligned}$$

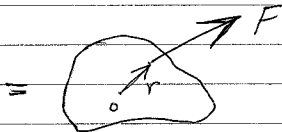
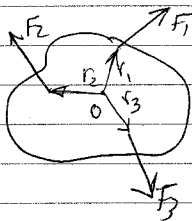
$$= (r_x F_z - r_z F_y) \hat{i} - (r_x F_z - r_z F_x) \hat{j} + (r_y F_x - r_x F_y) \hat{k}$$



$r$  = the position vector from the point you want to calculate the moment about to any point in the line of action of the force.

$$\text{magnitude} = |M_o| = |\vec{r} \times \vec{F}| = |r| |F| \sin \theta = Fd, \quad d = \text{shortest distance}$$

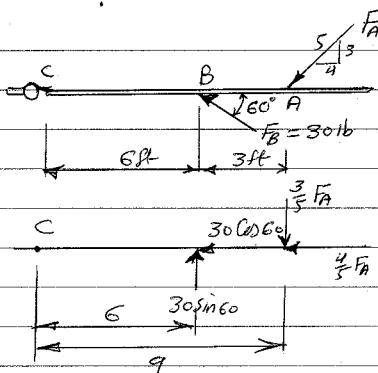
③ The moment of a set of forces equal to the sum of the moments of the individual force or the moment of their resultant.



$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$M_o = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{r}_3 \times \vec{F}_3 = \vec{r} \times \vec{F}$$

25  
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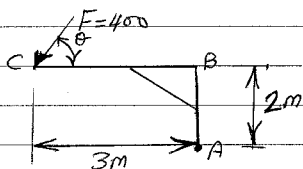
Determine  $F_A$  such that the total moment about C equal to zero.

$$\sum M_C = 0 + \uparrow$$

$$30 \sin 60 (6) - \frac{3}{5} F_A (9) = 0$$

$$F_A = 28.91b$$

4.3  
133



Determine the maximum and minimum moment of  $F$  about point A (specify  $0 \leq \theta \leq 80^\circ$ ).

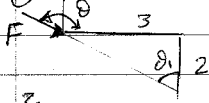
- ① Max. Moment occurs when the force has the largest distance from A to be  $d$ , which means  $F \perp AC$



$$\theta = \tan^{-1}\left(\frac{3}{2}\right) \Rightarrow \theta = 56.3^\circ$$

$$M = Fd = 400 (\sqrt{2^2 + 3^2}) = \underline{1442.4 \text{ N}\cdot\text{m}}$$

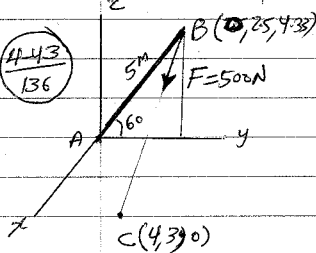
- ② Min moment occurs when the force  $F$  passes through A.



$$\theta = 0, +90 = 56.3 + 90 = 146.3^\circ$$

$$M = 0$$

4.43  
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Determine the moment of  $F$  about A and the shortest distance

$$\vec{M}_A = \vec{r}_{AC} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & 0 \\ 338.07 & 42.85 & -365.9 \end{vmatrix}$$

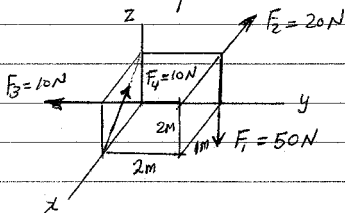
$$= -1098\hat{i} + 1464\hat{j} - 845\hat{k}$$

$$|\vec{M}_A| = \sqrt{1098^2 + 1464^2 + (-845)^2} = 2016 \text{ N}\cdot\text{m}$$

$$|\vec{M}_A| = Fd = 500d \Rightarrow d = \frac{M}{F} = \frac{2016}{500} = 4.03 \text{ m}$$

$$F = 500 \frac{\vec{r}}{|\vec{r}|}$$

#### 4.5 Moment of a Force about a specified Axis



$$M_{axis} = F(\perp \text{ distance} = d)$$

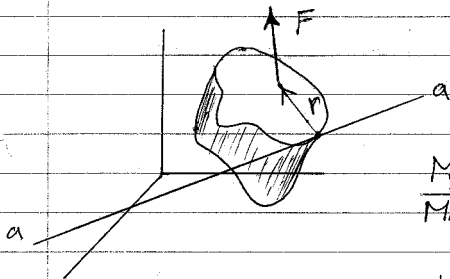
Moment of  $F_1$  about  $x$ -axis  $= -50(2)\hat{i} = -100\hat{i}$  (N-m)

" " " "  $y$ -axis  $= 0$  (intersect the  $y$ -axis)

" " " "  $z$ -axis  $= 0$  (parallel to the  $z$ -axis)

Moment of  $F_2$  about  $z$ -axis  $= 20(2)\hat{k} = 40\hat{k}$  (N-m)

Moment of  $F_3$  about  $y$ -axis  $= 0$   
 " " " "  $z$ -axis  $= 0$  } force intersect the axis  $y$  &  $z$ .

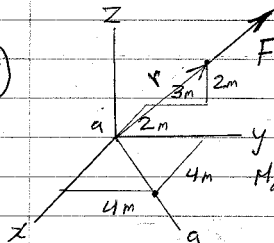


$$M_{a-a} = \vec{U}_{a-a} \cdot (\vec{r} \times \vec{F})$$

$$\vec{M}_{a-a} = [\vec{U}_{a-a} \cdot (\vec{r} \times \vec{F})] \vec{U}_{a-a}$$

$$\vec{U}_{a-a} \cdot (\vec{r} \times \vec{F}) = \begin{vmatrix} U_x & U_y & U_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

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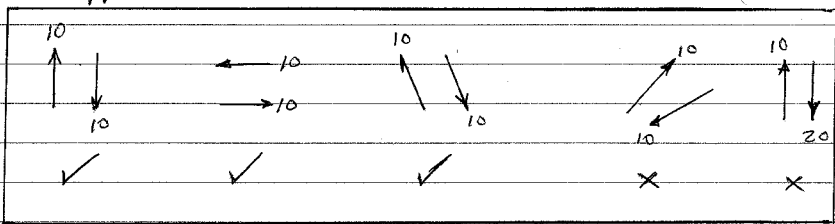
$$F = [30\hat{i} + 40\hat{j} + 20\hat{k}] \text{ N}, \quad \vec{r} = (-2, 3, 2), \quad \vec{U}_{a-a} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$$

$$M_{a-a} = \vec{U}_{a-a} \cdot (\vec{r} \times \vec{F}) = \begin{vmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -2 & 3 & -2 \\ 30 & 40 & 20 \end{vmatrix} =$$

$$= \frac{1}{\sqrt{2}}(140) + \frac{1}{\sqrt{2}}(90 + 80) = \frac{1}{\sqrt{2}}(310) = 219 \text{ N-m}$$

## 4.6 Moment of a couple

A couple is two forces equal in magnitude, parallel, and opposite in direction.



Moment of a couple is a free vector (i.e. independent of the point you choose).

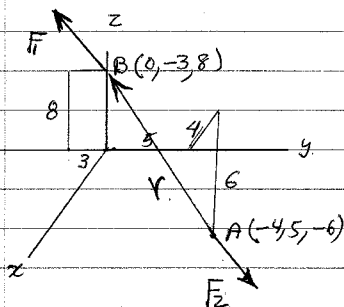
$$M_c = Fd$$

$$M = \mathbf{r} \times \mathbf{F}_1 \quad \text{not } (\mathbf{r} \times \mathbf{F}_2) \quad [\mathbf{F}_1 = \mathbf{F}_2]$$

$\mathbf{r}$  = position vector going from one force to the other

$\mathbf{F}$  = the force where the position vector is going to ( $\mathbf{F}_1$  not  $\mathbf{F}_2$ )

4.86



$$\mathbf{F}_1 = 50\mathbf{i} - 20\mathbf{j} + 80\mathbf{k}$$

$$\mathbf{F}_2 = -50\mathbf{i} + 20\mathbf{j} - 80\mathbf{k}$$

$$\mathbf{r} = 4\mathbf{i} - 8\mathbf{j} + 14\mathbf{k}$$

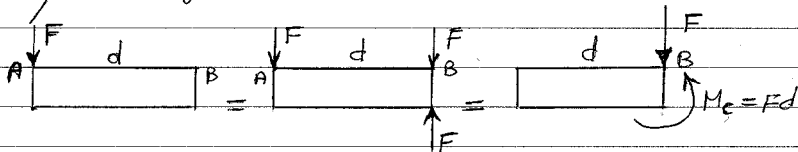
$$M_c = \mathbf{r} \times \mathbf{F}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -8 & 14 \\ 50 & -20 & 80 \end{vmatrix}$$

$$= \{360\mathbf{i} + 380\mathbf{j} + 320\mathbf{k}\}$$

N-m

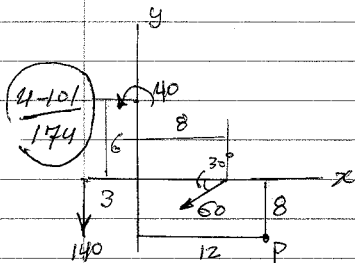
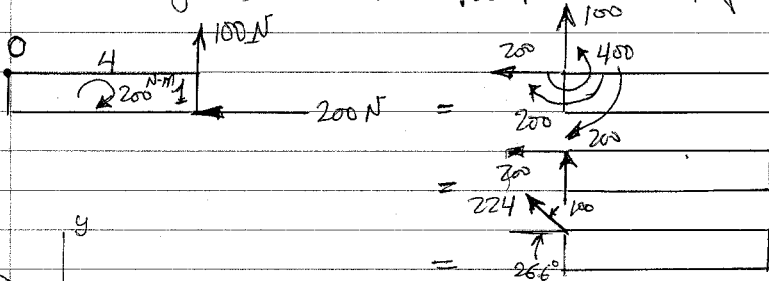


# 4.7 Equivalent System



i) When a force is moved from one point A to another point B, it will have the same force at B (same magnitude + direction) in addition to a couple moment at B  $\equiv Fd$ .

eg Replace the system of a forces and couple <sup>moment</sup> by a single force and couple moment at point O.

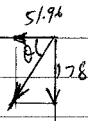


Single force:  $R_x = \sum F_x = -60 \cos 30^\circ = -51.96$

$R_y = \sum F_y = -140 - 60 \sin 30^\circ = -170$

$R = \sqrt{(51.96)^2 + 170^2} = 178 \text{ N}$

$\theta = \tan^{-1}\left(\frac{170}{51.96}\right) = 74^\circ$



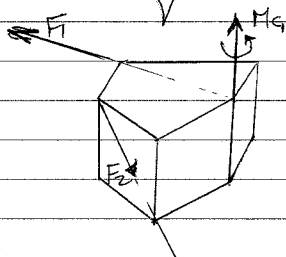
single moment  $M = 40 + [60 \sin 30^\circ][4] + [60 \cos 30^\circ][8] + 140(15)$

$M = 40 + 120 + 416 + 2100$

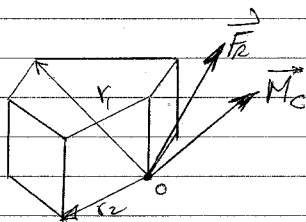
$= 2676 \text{ N-m}$

$= 2.68 \text{ kN-m}$

#### 4.8 Resultants of a force and couple System

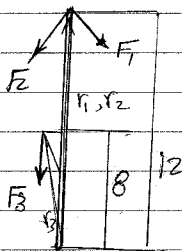


=

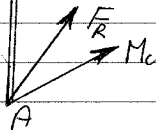


$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \dots$$

$$\vec{M}_0 = \vec{M}_0 + \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2$$



=



$$\begin{aligned}\vec{F}_1 &= 300\vec{i} + 400\vec{j} - 100\vec{k} \\ \vec{F}_2 &= 100\vec{i} - 100\vec{j} - 50\vec{k} \\ \vec{F}_3 &= \quad \quad -500\vec{k}\end{aligned}$$

$$\begin{aligned}\vec{r}_1 &= \vec{r}_2 = 12\vec{k} \\ \vec{r}_3 &= -\vec{j} + 8\vec{k}\end{aligned}$$

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \{400\vec{i} + 300\vec{j} - 650\vec{k}\} \text{ N}$$

$$M_0 = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{r}_3 \times \vec{F}_3 = \vec{r}_1 \times (\vec{F}_1 + \vec{F}_2) + \vec{r}_3 \times \vec{F}_3$$

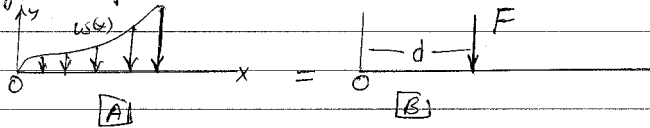
$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 12 \\ 400 & 300 & -150 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -1 & 8 \\ 0 & 0 & -500 \end{vmatrix}$$

$$= \{-3600\vec{i} + 4800\vec{j} + 0\vec{k}\} + \{500\vec{i} + 0\vec{j} + 0\vec{k}\}$$

$$= \{-3100\vec{i} + 4800\vec{j}\} \text{ N-m}$$

## 4-10 Reduction of a simple Distributed Loading

Two systems are equivalent if they produce the same force and moment about a given point

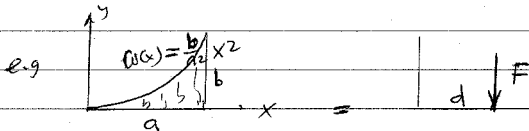


①  $F = \text{area under } w(x) \text{ curve} \Rightarrow$

$$F = \int w(x) dx$$

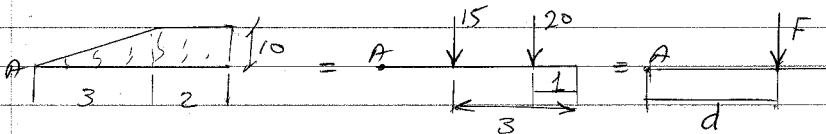
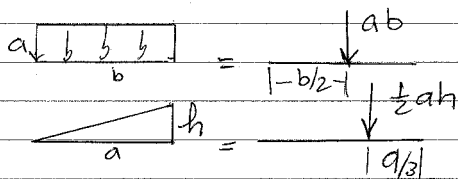
②  $(\text{moment of } w(x)) = (\text{moment of } F)$

$$\int x w(x) dx = F d \Rightarrow d = \frac{\int x w(x) dx}{\int w(x) dx}$$



$$F = \int_0^a w(x) dx = \int_0^a \frac{b}{a^2} x^2 dx = \frac{b}{a^2} \left[ \frac{x^3}{3} \right]_0^a = \frac{1}{3} ba$$

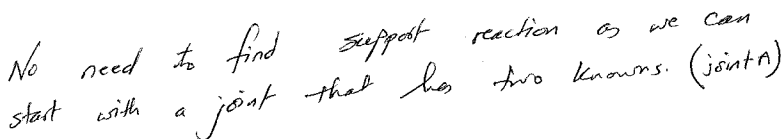
$$d = \frac{\int_0^a x \frac{b}{a^2} x^2 dx}{\frac{1}{3} ba} = \frac{\frac{b}{a^2} \int_0^a x^3 dx}{\frac{1}{3} ba} = \frac{\frac{1}{4} ba^2}{\frac{1}{3} ba} = \frac{3}{4} a$$



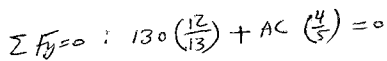
$$F = 20 + 15 = 35$$

$$M_A = -15(2) - 20(4) = -F d = -35 d \Rightarrow d = \frac{110}{35} = 3.14^m$$

Determine the force in each member of the truss.  
State if the force is tension (T) or compression



No need to find support reaction as we can start with a joint that has two knowns. (joint A)

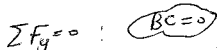


$$\sum F_y = 0 : 130 \left( \frac{12}{13} \right) + AC \left( \frac{4}{5} \right) = 0$$

$$\sum F_x = 0 : AB + AC\left(\frac{3}{5}\right) - BD\left(\frac{5}{13}\right) = 0$$

$$\sum F_x = 0 : AB + AC\left(\frac{3}{5}\right) - BD\left(\frac{5}{13}\right) = 0$$

$$AB = +140 \text{ lb}$$

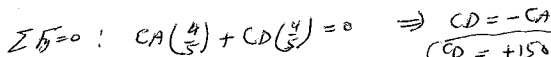


$$\sum F_x = 0; \quad BD - BA = 0$$

$$BD - (140) = 0$$

$$BD - (140) = 0$$

→  $BD = 140 \text{ lb}$



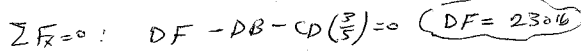
$$\sum F_x = 0: CA\left(\frac{4}{5}\right) + CD\left(\frac{4}{5}\right) = 0 \Rightarrow \frac{CD = -CA}{(CD = +150)}$$

$C_D = +150 \text{ lb}$

$$\sum F_x = 0: CE + \left(\frac{3}{5}\right)CD - CA\left(\frac{3}{5}\right) = 0$$

$$\sum_{k=0}^{\infty} : CE + \left(\frac{3}{5}\right)CD - CA\left(\frac{3}{5}\right) = 0$$

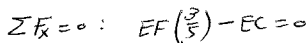
$$CE = -180^\circ$$



$$\sum F_x = 0: DF - DB - CD\left(\frac{3}{5}\right) = 0 \quad (DF = 23.016)$$

$$\sum F_y = 0: DE + DC \left( \frac{4}{5} \right) = 0 \quad (DE = -120 \text{ lb})$$

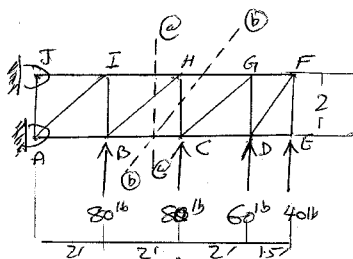
$$\sum F_y = 0: DE + DC \left( \frac{4}{5} \right) = 0 \quad DE = -120 \text{ lb}$$



$$\sum F_x = 0 : EF\left(\frac{3}{5}\right) - EC = 0$$

$$EF = -300/b$$

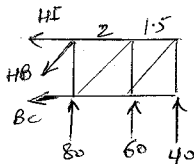
member	Force (lb)
AB	140 (T)
AC	150 (C)
CB	0
BD	140 (T)
DF	230 (T)
CE	180 (C)
CD	150 (T)
ED	120 (C)
EF	300 (C)



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- ① Using right hand side of section (a-a)  $\sum F_y = 0$  (solve for BH)  
 $\sum F_x = 0$  (solve for BC)
- ② Using right hand side of section (b-b)  $\sum F_y = 0$  (solve for HC)

Consider section (a-a)



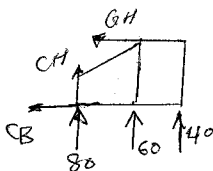
$$\sum F_y = 0 \quad 40 + 60 + 80 - HB \frac{1}{\sqrt{2}} = 0$$

$$HB = 255 \text{ T}$$

$$\sum M_{@H} = 0 \quad 40(35) + 60(2) = BC(2)$$

$$BC = 130 \text{ T}$$

Consider section (b-b)



$$\sum F_y = 0 \quad 40 + 60 + 80 + CH = 0$$

$$CH = 180 \text{ C}$$

## **Frames & Machine Analysis**

- (1) Draw free body diagram of the whole structure, showing all the applied loads and reactions.
- (2) Count number of external unknown reactions and compare them with the total number of equations of equilibrium (normally three questions) if number of reactions more than number of equations (greater than 3) then one need to disconnect members. Otherwise apply equations of equilibrium and find the unknowns.
- (3) Before disconnecting, if there is pulley attached to the frame, disconnect the pulley and find its pin reactions and put these reactions on the structure where the pulley was connected as applied load in the opposite direction.
- (4) Disconnect members and draw F.B.D of each member and of any joint which has load applied to it. Start naming reactions with the two force members (remember disconnection means equal forces and opposite direction).
- (5) Apply again equations of equilibrium to members and joints separately. Remember there are three equations of equilibrium for each member (a non two-force member) and two equations of equilibrium for each joint.

If total number of equations of equilibrium equal to the total number of unknowns then proceeding to solve for the unknown, otherwise you must have made a mistake.

**Note:** When two members meet at a joint where load is acting then their forces are independent and the relationship will come through the equilibrium of the joint.