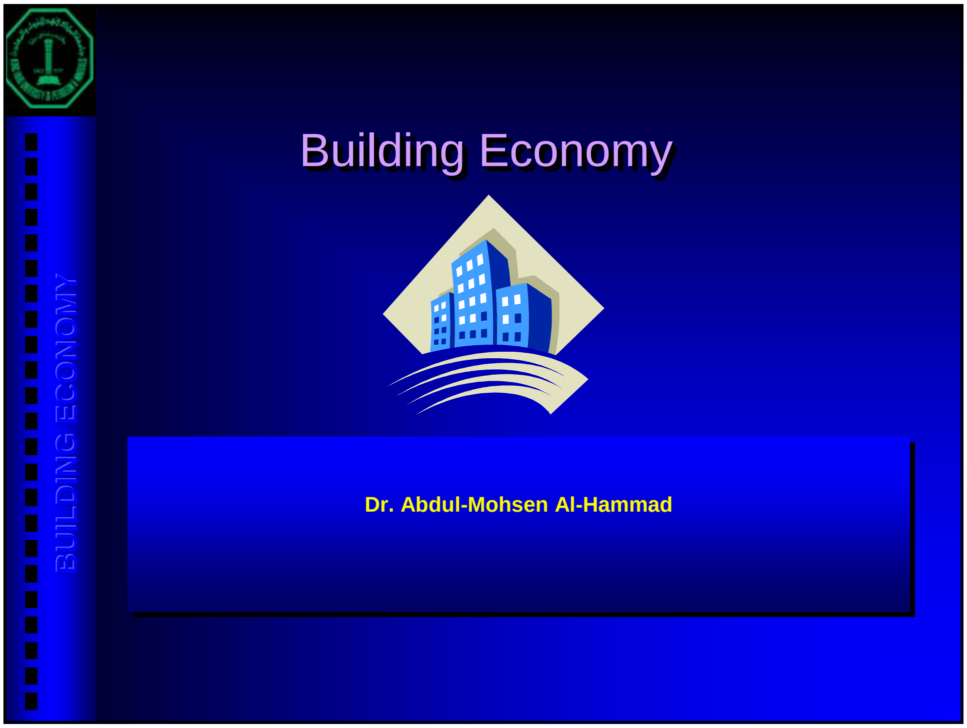
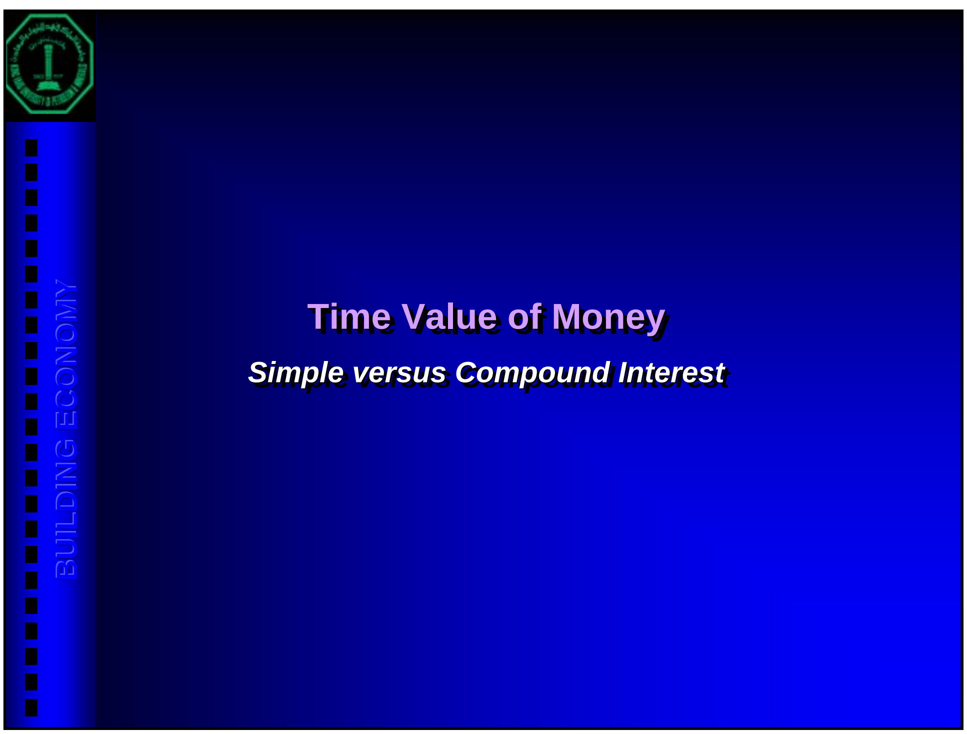

Building Economy
BUILDING ECONOMY
Dr. Abdul-Mohsen Al-Hammad


BUILDING ECONOMY
Time Value of Money
Simple versus Compound Interest

Outline

Simple Interest.

Example Illustrating Simple Interest Calculations.

Compound Interest.

Example Illustrating Compound Interest Calculations.

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3

Simple versus Compound Interest 1/8

Simple interest is calculated using the principal only (i.e. the original investment or original loan), ignoring any interest that has been accrued in preceding interest periods.

Simple Interest = Principal (P) X Number of periods (n) X Interest rate (i)

Simple Interest = (P)(n)(i)

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Simple versus Compound Interest 2/8

Example: **If you borrow $1,000 for three years at 6% per year simple interest, how much money will you owe at the end of three years?**

Simple Interest = Principal (P) X Number of periods (n) X Interest rate (i)

Simple Interest = $1,000 X 3 X 0.06 = $180

Amount due after three years = $1,000 + $180 = $1,180

Simple interest is seldom used in today's economics.

5

Simple versus Compound Interest 3/8

Compound interest is calculated using the principal plus the total amount of interest accumulated in previous periods.

Thus, compound interest means "interest on top of interest".

Example: If you borrow $1,000 at 6% per year compound interest, compute the total amount owed after three year period?

6

Simple versus Compound Interest 4/8

Interest for Year 1 = \$1,000 X 0.06 = \$60

Total amount due after year 1 = \$1,000 + \$60 = \$1,060

Interest for year 2 = \$1,060 X 0.06 = \$63.60

Total amount due after year 2 = \$1,060 + \$63,60 = \$1,123.60

Interest for year 3 = \$1,123.60 X 0.06 = \$67.42

Total amount due after year 3 = \$1,123.60 + \$67.42 = \$1,191.02

Thus, with compound interest, the original \$1,000 would accumulate an extra \$1,191.02 - \$1,180 = \$11.02 compared to simple interest in the three year period.

7

Simple versus Compound Interest 5/8

Therefore, if an amount of money P is invested at some time t=0, the total amount of money (F) that would be accumulated after one year would be:

$F_1 = P + Pi$

$F_1 = P(1 + i)$

Where F_1 = The total amount accumulated after one year

8

Simple versus Compound Interest 6/8

At the end of the second year, the total amount of money accumulated (F_2) would be equal to the total amount that had accumulated after year 1 plus interest from the end of year 1 to the end of year 2.

$F_2 = F_1 + F_1 i$

$F_2 = P(1 + i) + P(1 + i)i$

$F_2 = P(1 + i + i + i^2)$

$F_2 = P(1 + 2i + i^2)$

$F_2 = P(1 + i)^2$

9

Simple versus Compound Interest 7/8

Similarly, the total amount of money accumulated at the end of year 3 (F_3) would be equal to the total amount that had accumulated after year 2 plus interest from the end of year 2 to the end of year 3.

$F_3 = F_2 + F_2 i$

$F_3 = \left[P(1 + i) + P(1 + i)\, i\right] + \left[P(1 + i) + P(1 + i)\, i\right] i$

Factoring out $P(1 + i)$ we have:

$F_3 = P(1+ i)\,(1 + 2i + i^2)$

$F_3 = P(1 + i)(1 + i)^2$

$F_3 = P(1 + i)^3$

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Simple versus Compound Interest 8/8

From the preceding values, it is evident that by mathematical induction that the formula for calculating the total amount of money (F) after (n) number of years, using compound interest (i) would be:

$$F_n = P(1 + i)^n$$

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Time Value of Money

Cash Flow/Time Diagrams

Outline

- The Concept of Cash Flow.
- Cash Flow Tabulation.
- Examples Illustrating Cash Flow Tabulations.
- Cash Flow Diagrams.
- Example Illustrating the Use of Cash Flow Diagrams.

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Symbols and Cash Flow Diagrams

The mathematical relations used in engineering economy employ the following symbols:

Note:

The dollar amount of F and A are considered at the end of the interest period.

- **P** = Value of sum of money at a time denoted as the present.
- **F** = Value or sum of money at some future time, or a single sum of money at the end of n interest period.
- **A** = A series of periodic, equal amount of money.
- **n** = Number of interest periods.
- **i** = Interest rate per interest period.

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Cash Flow 1/3

Every person or company has cash receipts (income) and cash disbursement (costs).

The results of income and costs is called cash flow.

Cash Flow = Receipts – Disbursements

A positive cash flow indicates a net receipts in a particular interest period or year.

A negative cash flow indicates a net disbursement in that period.

15

Cash Flow 2/3

Example: If you buy a printer in 1999 for $300, maintain it for three years at a cost of $20 per year, and then sell it for $50, what are your cash flows for each year?

Year	Receipts	Disbursement	Cash Flow
1999	0	$300	- $300
2000	0	$20	- $20
2001	0	$20	- $20
2002	$50	$20	+ $30

Its important to remember that all receipts and disbursements and thus cash flows are assumed to be end-of period amounts. Therefore, 1999 is the present (now) and 2002 is the end of year 3.

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Cash Flow 3/3

Example: Suppose you borrowed $1,000 on May 1, 1984, and agree to repay the loan in one lump sum of $1,402.60 at the end of four years at 7%. Tabulate the cash flows?

Date	Receipts	Disbursement	Cash Flow
May 1, 1984	$1,000	0	+ $1,000
May 1, 1985	0	0	0
May 1, 1986	0	0	0
May 1, 1987	0	0	0
May 1, 1988	0	$1,402.60	- $1,402.60

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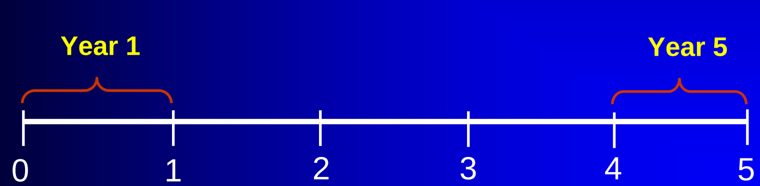
Cash Flow Diagrams 1/6

A cash flow diagram is simply a graphical representation of cash flows (in vertical direction) on a time scale (in horizontal direction). Time zero is considered to be present, and time 1 is the end of time period 1.

This cash flow diagram is set up for five years.



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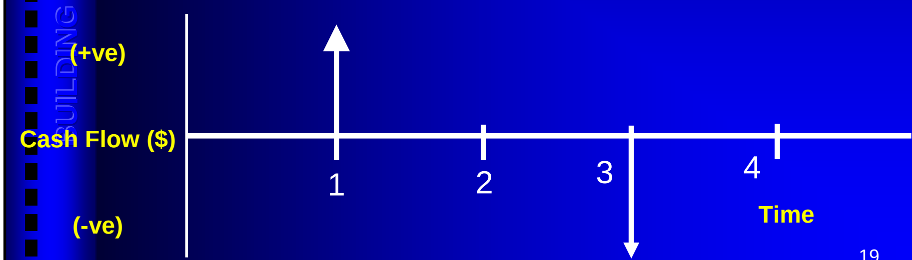
Cash Flow Diagrams 2/6

The direction of the cash flows (income or outgo) is indicated by the direction of the arrows.

From the investor's point of view, the borrowed funds are cash flows entering the system, while the debt repayments are cash flows leaving the system.



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Cash Flow Diagrams 3/6

Cash Flow Diagrams Benefit

- It helps to clarify one's viewpoint for economic analysis
- It requires a clear definition of the economic system
- It focuses attention on cash flow between the system and parties external to the system.
- It is an efficient and unequivocal method for communicating all cash flow information utilized in an economic analysis.
- It reduces errors in interest computation analysis.

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Cash Flow Diagrams 3/6

Example 1: **If you borrow $2,000 now and must repay the loan plus interest (at rate of 6% per year) after five years. Draw the cash flow diagram. What is the total amount you must pay?**

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Cash Flow Diagrams 3/6

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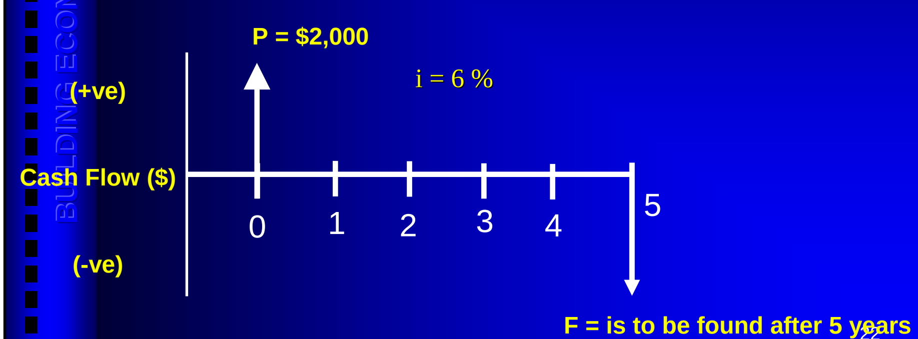

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Cash Flow Diagrams 4/6

Example 2: If you start now and make five deposits of $1,000 per year (A) in a 7% per year account, how much money will be accumulated immediately after you have made the last deposit. Draw the cash flow diagram. What is the total amount you will accumulate?

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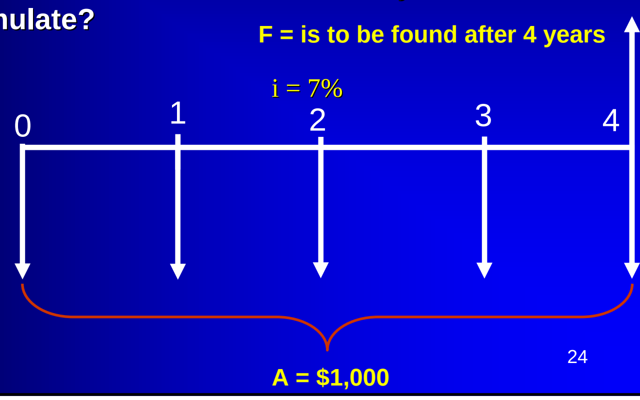
Cash Flow Diagrams 4/6

Example 2: If you start now and make five deposits of $1,000 per year (A) in a 7% per year account, how much money will be accumulated immediately after you have made the last deposit. Draw the cash flow diagram. What is the total amount you will accumulate?

Since you have decided to start now, the first deposit is at year zero and the fifth deposit and withdrawal occur at end of year 4



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Cash Flow Diagrams 5/6

BUILDING ECONOMY

Example 3: **Assume that you want to deposit an amount (P) into an account two years from now in order to be able to withdraw $400 per year for five years starting three years from now. Assume that the interest rate is 5.5% per year. Construct the cash flow diagram.**

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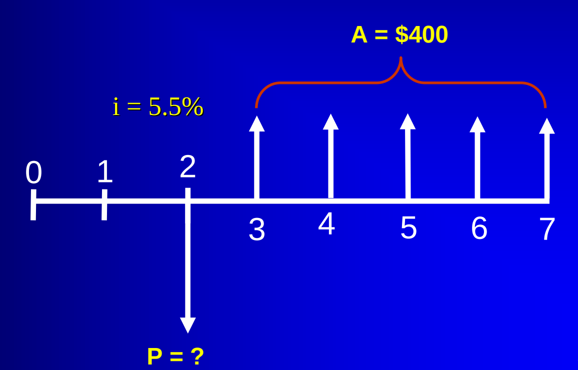
Cash Flow Diagrams 5/6

BUILDING ECONOMY

Example 3: **Assume that you want to deposit an amount (P) into an account two years from now in order to be able to withdraw $400 per year for five years starting three years from now. Assume that the interest rate is 5.5% per year. Construct the cash flow diagram.**



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Cash Flow Diagrams 6/6

BUILDING ECONOMY

Example 4: **Suppose that you want to make a deposit into your account now such that you can withdraw an equal amount (A1) of $200 per year for the first five years starting one year after your deposit and a different annual amount (A2) of $300 per year for the following three years. With an interest rate (i) of 4.5% per year, construct the cash flow diagram.**

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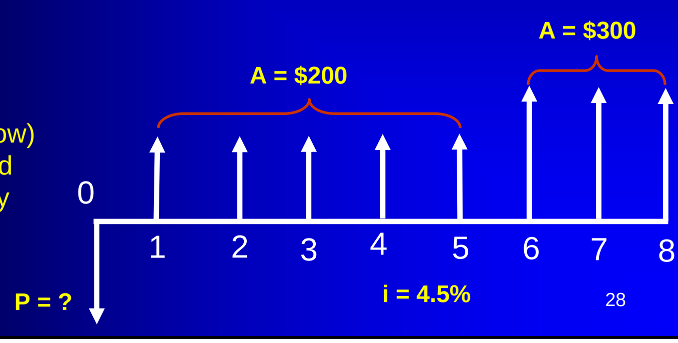
Cash Flow Diagrams 6/6

BUILDING ECONOMY

Example 4: **Suppose that you want to make a deposit into your account now such that you can withdraw an equal amount (A1) of $200 per year for the first five years starting one year after your deposit and a different annual amount (A2) of $300 per year for the following three years. With an interest rate (i) of 4.5% per year, construct the cash flow diagram.**

The first withdrawal (positive cash flow) occurs at the end of year 1, exactly one year after P is deposited.



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Time Value of Money

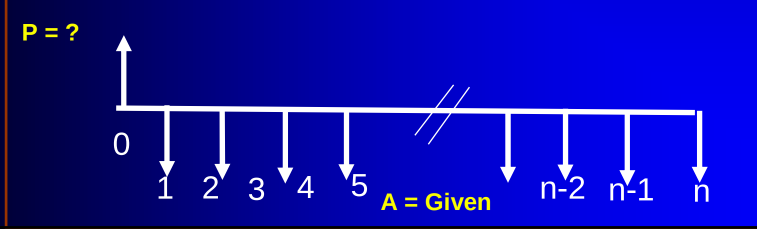
Interest Factors and Equations

Interest Factors and Equations 1/3

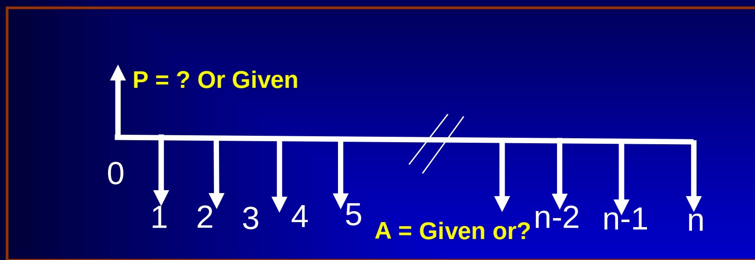
$$F_n = P(1 + i)^n$$

$$P = F \frac{1}{(1 + i)^n}$$

Derivation of the Uniform-series Present – worth factor and the Capital – Recovery Factor



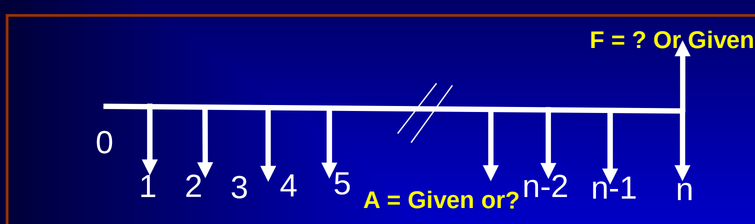
Interest Factors and Equations 2/3



After the derivation

$$A = P \frac{i(1+i)^n}{(1+i)^n - 1} \quad , \quad P = A \frac{(1+i)^n - 1}{i(1+i)^n} \quad ,$$

Interest Factors and Equations 3/3



After the derivation (Continue)

$$A = F \frac{i}{(1+i)^n - 1} \quad , \quad F = A \frac{(1+i)^n - 1}{i}$$

BUILDING ECONOMY

Time Value of Money

Use of Interest Tables

BUILDING ECONOMY

Outline

Interest Factor Names and Standard Notations.

Example Illustrating the Use of Interest Factors.

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Use of Interest Tables

To avoid the trouble of writing out each formula, a standard notation is used:

(X/Y, i%, n)

The first letter in the parentheses (X) represent what you "Want to find", while the second letter (Y) represents what is "Given".

For example, F/P means "find F when given P". The i is the interest rate in percent and n represents the number of periods involved.

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Use of Interest Tables 1/8

Factor Name	Standard Notation
Single-Payment Present Worth Factor	(P/F, i%, n)
Single-Payment Compound-Amount Factor	(F/P, i%, n)
Uniform Series Present Worth Factor	(P/A, i%, n)
Capital Recovery Factor	(A/P, i%, n)
Sinking Fund Factor	(A/F, i%, n)
Uniform-Series Compound Amount Factor	(F/A, i%, n)

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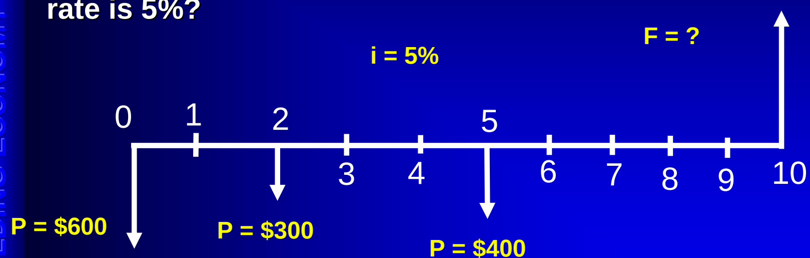
Use of Interest Tables 2/8

Example 1: If a person deposits \$600 now, and \$300 two year from now, and \$400 five years from now, how much will be have in his account ten years from now if the interest rate is 5%?



F = \$600(F/P, 5%, 10) + \$300(F/P, 5%, 8) + \$400(F/P, 5%, 5)
F = \$600(1.6289) + \$300(1.4774) + \$400(1.2763)
F = \$1931.08

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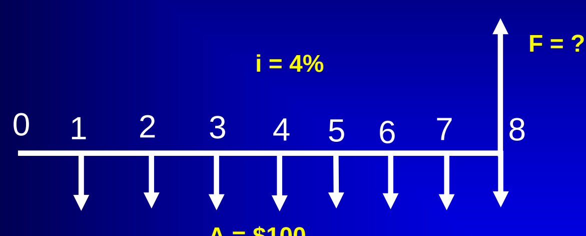
Use of Interest Tables 3/8

Example 2: How much money would a person have in his account after eight years if he deposited \$100 per year for eight years at 4% starting one year from now?



F = \$100(F/A, 4%, 8)
F = \$100(9.214)
F = \$921.40

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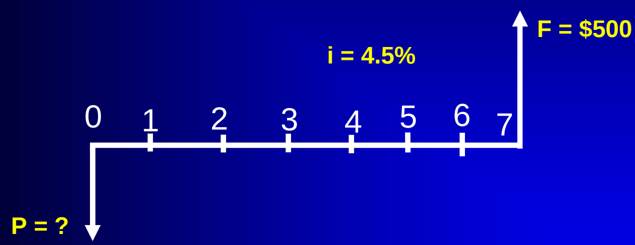
Use of Interest Tables 4/8

Example 3: **How much money would you be willing to spend now in order to avoid spending $500 seven years from now if the interest rate is 4.5%?**



F = $500(P/F, 4.5%, 7)

F = $500(0.7353)

F = $367.65

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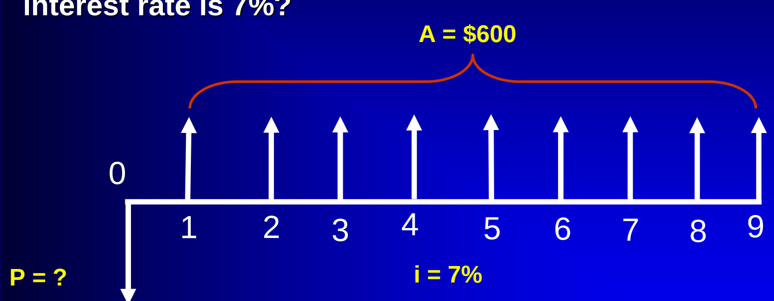
Use of Interest Tables 5/8

Example 4: **How much money would you be willing to pay now for a note that will yield $600 per year for nine years if the interest rate is 7%?**



P = $600(P/A, 7%, 9)

P = $600(6.5152)

P = $3909.12

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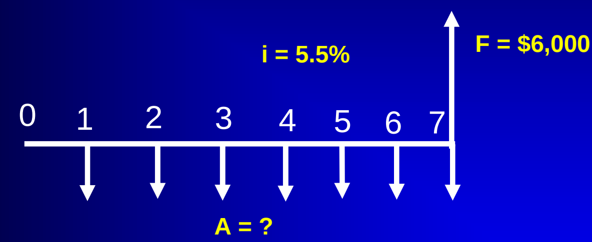
Use of Interest Tables 6/8

Example 5: **How much money must a person deposit every year starting one year from now at 5.5% per year in order to accumulate $6,000 seven years from now?**



A = $6,000(A/F, 5.5%, 7)

A = $6,000(0.12098)

A = $725.88 per year

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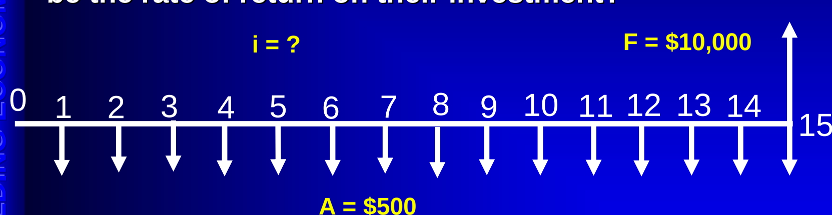
Use of Interest Tables 7/8

Example 6: **A couple wishing to save money for their child's education purchased an insurance policy that will yield $10,000 15 years from now. The parent must pay $500 per year for the 15 years starting one year from now. What will be the rate of return on their investment?**



A = F(A/F, i%, n) → $500 = $10,000(A/F, i%, 15) → (A/F, i%, 15) = 0.0500

From the interest tables under the A/F column for 15 years, the value of 0.0500 is found to lie between 3% and 4%.
By interpolation, i = 3.98%

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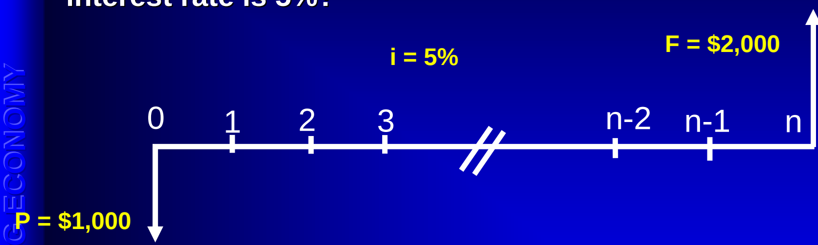
Use of Interest Tables 8/8

Example 7: **How long would it take for $1,000 to double if the interest rate is 5%?**



P = F(P/F, i%, n)

$1000 = $2,000(P/F, 5%, n)

(P/F, 5%, n) = 0.500

From the 5% interest table, the value of 0.500 under the P/F column lies between 14 and 15 years.
By interpolation, n = 14.2 years

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BUILDING ECONOMY

Time Value of Money

Multiple Factors

BUILDING ECONOMY

Outline

Multiple Factors.

Example Illustrating the Use of Multiple Factors.

Calculations Involving Uniform-Series and Randomly Distributed Amounts

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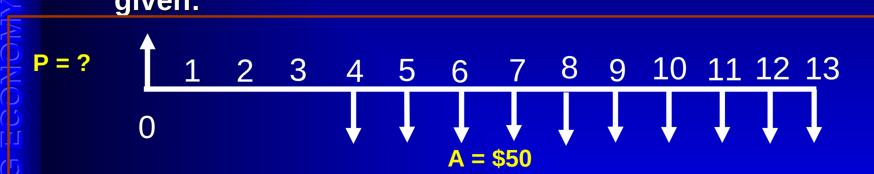
Multiple Factors 1/10

When a uniform series of payment (A) begins at a time other than the end of year 1, several methods can be used to find the present worth (P). For example, given:



We can use the following methods:

1. **Use the single-payment present worth factor (P/F, i%, n) to find the present worth of each disbursement at year Zero, then add them.**

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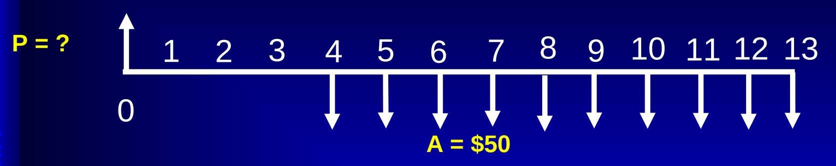
Multiple Factors 2/10



2. Use the single-payment compound-amount factor (F/P, i%, n) to find the future worth of each disbursement in year 13, add them, and then find the present worth of the total using P = F(P/F, i%, 13).

3. Use the uniform-series compound-amount factor (F/A, i%, n) to find the future amount by F = A(F/A, i%, 10) and then find the present worth using P = F(P/F, i%,13).

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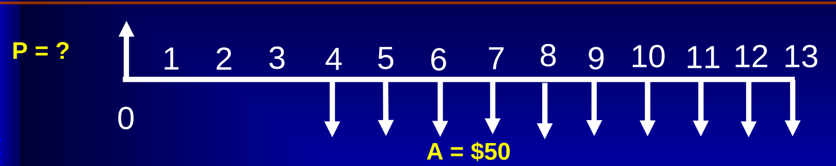
Multiple Factors 3/10



4. Use the uniform-series present-worth factor (P/A, i%, n) to compute the present worth at year 3 and then find the present worth in year Zero by using the (P/F, i%, n) factor.

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Multiple Factors 4/10

Note:

It is very important to remember that the present worth is always located One year prior to the first annual payment when using the uniform-series present-worth factor (P/A, i%, n).

On the other hand, the uniform-series compound-amount factor (F/A, i%, n) was derived with the future worth "F" located in the same year as the last payment.

It is always important to remember that the number of years n that should be used with the P/A or F/A factors is equal to the number of payments. It is generally helpful to re-number the cash-flow diagram to avoid counting errors.

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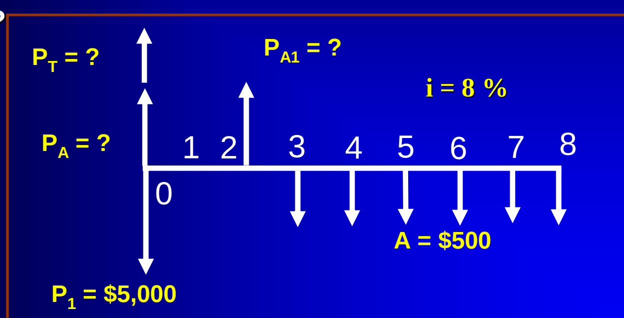
Multiple Factors 5/10

Example 1: A person buys a piece of property for $5,000 down-payment and deferred annual payments of $500 a year for six years starting three years from now. What is present worth of the investment if the interest rate is 8%?



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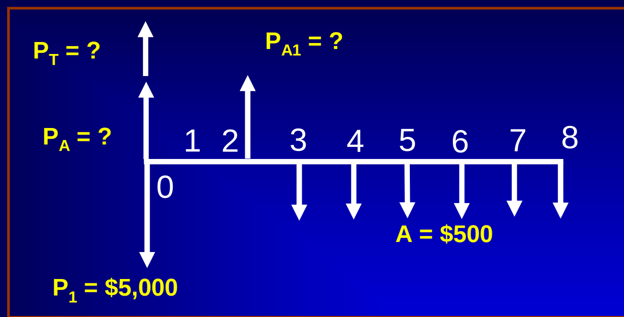
Multiple Factors 6/10



P_{A1} = The present worth of a uniform-series.

P_A = The present worth at a time other than zero.

P_T = Total present worth at time zero.

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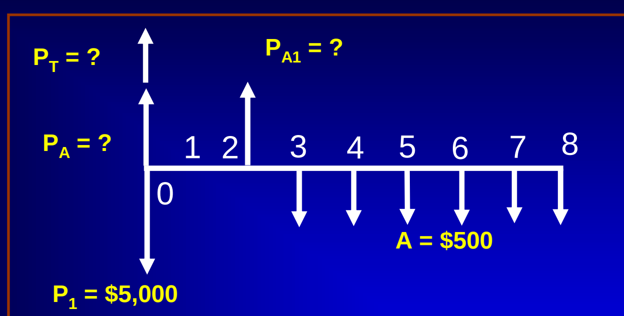
Multiple Factors 7/10



P_{A1} = 500 (P/A, 8%, 6)

$P_A = P_{A1}$ (P/F, 8%, 2)

$P_T = P_1 + P_A$

P_T = \$5,000 + \$500 (P/A, 8%, 2) = \$5,000 + \$500 (0.8573)

P_T = \$6,981.60

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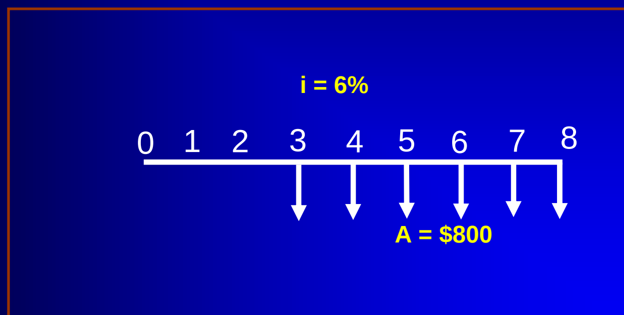
Multiple Factors 8/10

Example 2: Calculate the eight-year equivalent uniform annual series at 6% interest for the uniform disbursements shown in the figure below.



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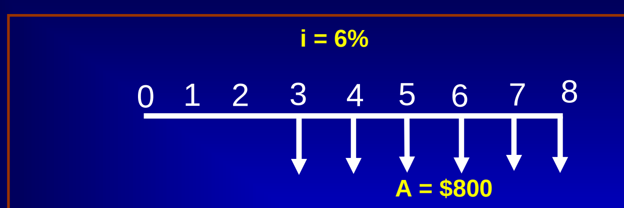
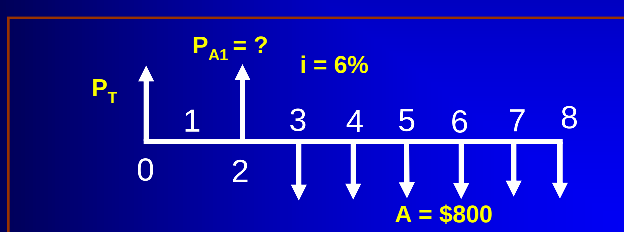
Multiple Factors 9/10





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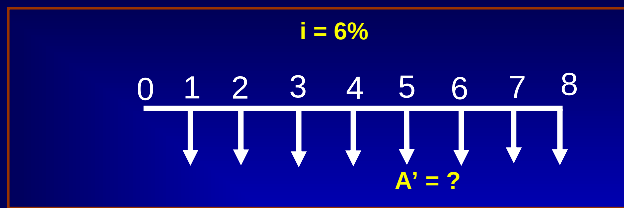
Multiple Factors 10/10



P_{A1} = 800 (P/A, 6%, 6)

P_T = P_{A1} (P/F, 6%, 2) = $3,501.12

A' = P_T(A/P, 6%, 8)

A' = $563.82

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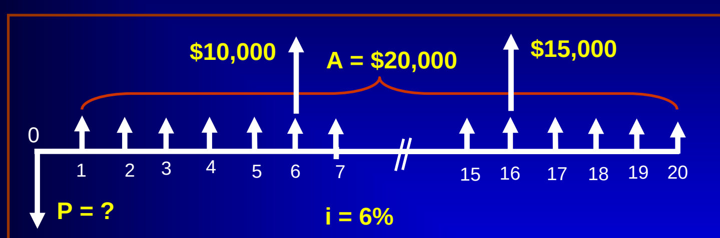
Calculations Involving Uniform-Series and Randomly Distributed Amounts- 1/5

Example3: Calculate the present worth (P) ?



Find the present worth of the uniform-series and add it to the present-worth of the two individual payments:

P = $20,000 (P/A, 6%, 20) + $10,000 (P/F, 6%, 6) + $15,000 (P/F, 6%, 16)

P = $242,352

BUILDING ECONOMY

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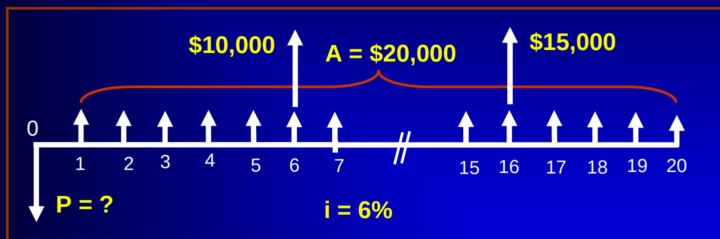
Calculations Involving Uniform-Series and Randomly Distributed Amounts- 2/5

Example 4: Calculate the equivalent uniform annual series (A)?



Two methods could be used:

a. Present worth method.
b. Future worth method.

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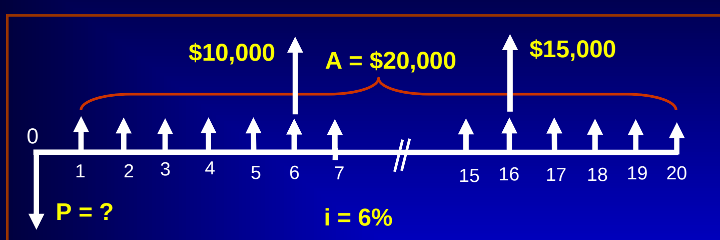
Calculations Involving Uniform-Series and Randomly Distributed Amounts- 3/5



Present worth method:

A = $20,000 + $10,000(P/F, 6%, 6) (A/P, 6%, 20) + $15,000(P/F, 6%, 16) (A/P, 6%, 20)

A = $21,129 per year

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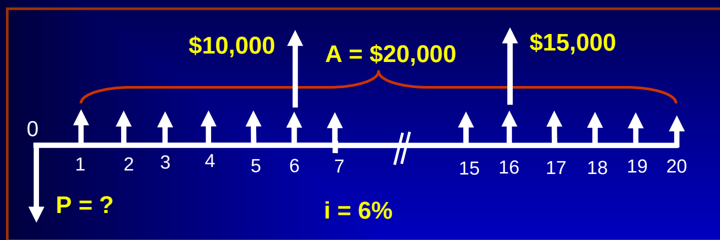
Calculations Involving Uniform-Series and Randomly Distributed Amounts- 4/5



Future worth method:

A = $20,000 + $10,000(F/P, 6%, 14) (A/F, 6%, 20) + $15,000(F/P, 6%, 4) (A/F, 6%, 20)

A = $21,129 per year

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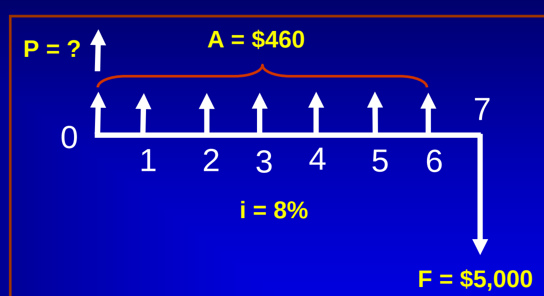
Calculations Involving Uniform-Series and Randomly Distributed Amounts- 5/5

Example 5: Calculate the present worth of the following series of cash flows if i = 8%?

Year	Cash Flow
0	+ $460
1	+ $460
2	+ $460
3	+ $460
4	+ $460
5	+ $460
6	+ $460
7	- $5,000



$P = P_1 + P_A – P_F$

P = $460 + $460(P/A, 8%, 6) – $5,000(P/F, 8%, 7)

P = -$331

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