An Approximate Model for Wave Propagation in Rectangular Rods and Their Geometric Limits

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(Received 3 December 1997; accepted 29 April 1998)

Abstract: An approximate model for wave propagation in rods of rectangular cross section was developed that is based on neglecting the dependence of the shear stresses and the longitudinal displacement on one, or both, of the in-cross-sectional coordinates. The resulting approximate system of equations, together with the appropriate boundary conditions, are then solved exactly, leading to a simple, but general, characteristic dispersion equation. The degenerate geometric limiting cases of infinite media and flat plates are obtained as special cases. The model also predicts all of the general features exhibited in Morse's experimental data (1948). These include the correct low- and high-frequency limits of the wave speeds, the cut-off frequencies, and the common point of crossing of the higher group of modes.

Key Words: Wave propagation, rectangular rods, analytical solution, dispersion

1. INTRODUCTION

Guided waves in rectangular plates and in rods of varying cross-sectional shapes have been extensively studied for more than a century. Original applications included delay lines for rods and noise insulators for plates. Modern applications also include modeling of composite materials and ultrasonic wave propagation for nondestructive evaluations and material characterization. Exact solutions are, however, available only for the simple geometries of infinite flat plates and circular rods. These solutions date back to the last century when Pochhammer (1876) and Chree (1889) derived exact dispersion formulas for rods and, early this century, where Lamb (1917) obtained those for plates. The relative ease in finding exact solutions, for these two systems, has been attributed to the fact that both are described with two-dimensional field equations. In more recent years, exact treatments have been extended to many cases of isotropic and anisotropic laminated and coaxial rod systems. An account of the individual contributions is well beyond this paper. However, for relevant literature, see, for example, Nayfeh (1995) and Auld (1990) for plate systems; Huang, Rokhlin, and Wang (1995) and Simmons, Drescher-Krasicka, and Wadley (1992) for isotropic coaxial rods; and Nayfeh and Nagy (1996) for anisotropic coaxial systems.

Parallel to the exact treatments, there have been as many (or even more) attempts to construct approximate models to capture the essential features embedded in the complicated