An improved continuum mixture model for wave propagation in fibrous composites

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A new improved continuum mixture model is developed for the propagation of axisymmetric longitudinal waves in fibrous composites. The major improvement on the original model of Hegemier, Gurtman, and Nayfeh [Int. J. Solids Struct. **9**, 395 (1973)] is achieved by the inclusion of the axial rate of change of the radial displacement in the shear constitutive relations which was neglected previously. This model has also been extended to treat situations in which the fiber and the matrix are anisotropic. The improved model is found superior to the original one, when compared with the recently acquired experimental data and exact solutions. © *1998 Acoustical Society of America.* [S0001-4966(98)00508-6]

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INTRODUCTION

In a recent paper,[1] Huang and Rokhlin presented low-frequency experimental dispersion data for longitudinal wave propagation in a fibrous composite system. The specific composite tested consisted of a titanium alloy matrix reinforced with three-layer concentric cylindrical SiC fibers (SCS-6). Each fiber is about 140 μm in total diameter and consists of a carbon core and a SiC shell separated from the matrix by another thin 3-μm-thick carbon layer. All fiber layers, as well as the matrix, are isotropic materials. Huang and Rokhlin compared this experimental data with the available continuum mixture theory model of Hegemier *et al.*[2] Comparisons were shown to be good at arbitrary low-frequency ranges. At relatively higher frequencies, the discrepancy increases significantly and it was suggested that better agreement with the exact solutions was expected. In a subsequent paper,[3] Huang and Rokhlin derived an exact dispersion relation for a multilayered concentric cylindrical fiber reinforced composite. In their study, all of the fiber components as well as the matrix are assumed isotropic. Concurrent with their study, Nayfeh and Nagy[4] derived exact dispersion relations for a similar system when all individual composite layers are anisotropic. Both of the exact solutions were found to better fit the experimental data.

Due to the comparative simplicity, and ease in utility, of the approximate mixture modeling, we have decided to examine the reason for its inability to better fit the data especially at the moderately higher-frequency ranges. We do this with the intention of improving the previously constructed model in Ref. 2. Before we proceed to derive the new improved continuum mixture model, it is instructive to review the various assumptions and approximations adopted in constructing the original model.

In the original continuum mixture theory, fibers made up of single isotropic material are uniformly distributed in an isotropic material matrix. The resulting hexagonal symmetry of the unidirectionally reinforced composite permitted the isolation of a representative unit cell, which in turn was modeled as a concentric cylinder subjected at its outer boundary to vanishing radial displacement and shear stress. Next, guided by the various symmetries and fiber-matrix interface conditions, certain approximate radial dependencies of some of the field variables are assumed which satisfy these conditions. The only other critical approximation was to neglect the term representing the axial rate of change of the radial displacement from the shear constitutive relation for both the fiber and the matrix. As a consequence, the two-dimensional field equations that hold in both the fiber and the matrix, together with their interface continuity conditions, are reduced to a quasi-one-dimensional system of two coupled partial differential equations that automatically satisfy all interface and radial boundary conditions. By assuming a harmonic solution for the field variables, the characteristic dispersion equation of the system is obtained. This characteristic equation predicts the existence of two modes. The fundamental one, which is the only mode that can propagate at arbitrarily low frequencies, starts from the mixture speed at the zero frequency limit and continuously varies to the high-frequency limit of the bulk wave speed in the matrix. The second mode appears after its cutoff frequency and converges, at high frequencies, to the bulk wave speed in the fiber.

In the present work, we develop an improved mixture model based on retaining the term representing the axial rate of change of the radial displacement, which was neglected in the previous model. Since the continuum mixture solutions are most appropriate for two homogenous phase media, namely for media consisting of single material fiber and matrix, we need to model the multilayered fiber of Ref. 1, as an effective homogenized one. Due to the biased geometry of the fiber layers, the effective fiber will have anisotropic properties or, more precisely, will exhibit transverse isotropy. The effective anisotropic properties of the three-component fiber tested in Ref. 1 will be constructed by a repetitive use of the procedure developed by Nayfeh[5] for deriving effective properties for two-component fibrous composites. Thus, the present situation dictates that the applicable continuum mixture model has to be modified for anisotropic media. For this